系所組別： 環境工程學系丙組
考試科目： 微積分   考試日期：0219，節次：3

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1. Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour, recorded in the following table. (16 points)

Time (hour)	0	1	2	3	4	5	6	7	8
Leakage (gallon/hour)	50	70	97	136	190	265	369	516	720

1-1) Give an upper and lower estimate of the total quantity of oil that has escaped after 5 hours using upper and lower summation. (8)

1-2) It is known that the error in Trapezoidal rule is twice as much as that of Midpoint rule, and the error is always the opposite. (When Trapezoidal rule has positive error, Midpoint rule has negative error.) Use this information to obtain the best estimate of the oil that has escaped in 8 hours. (8)
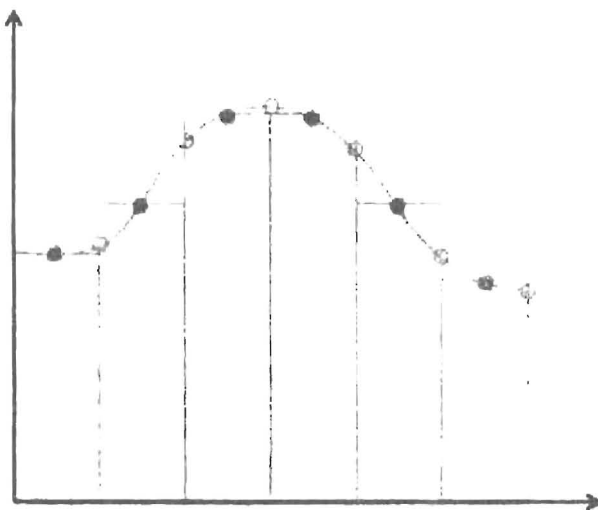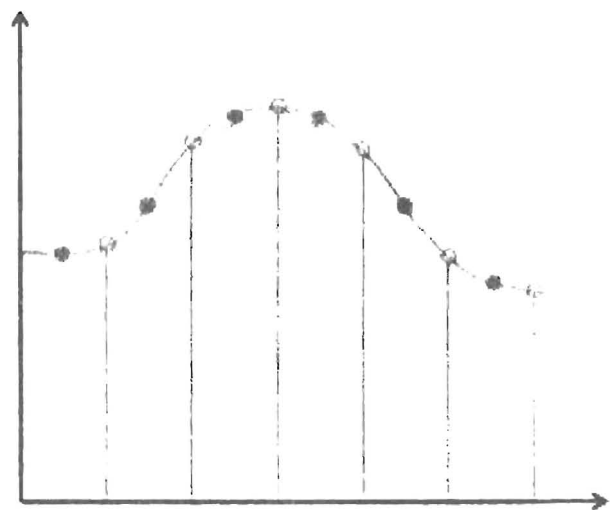
Midpoint rule   *Trapezoidal rule*

2. The plane $x+y+z=1$ cuts the cylinder $x^2+y^2=1$ in an ellipse. Find the points on the ellipse that lie closest to and farest from the origin. (12 points)

3. By using the fact that $\log n \ll n$, evaluate $\lim\limits_{n \to \infty} n^{1/n}$. (10 points)

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4. Answer the following question. (30 points)

4-1) Provide Maclaurin series of $\sin x$ and $\cos x$. **(6)**

4-2) Calculate $\displaystyle\lim_{x\to0}\frac{1-\cos x}{x\sin x}$ **(8)**

4-3) Calculate $\displaystyle\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n$ **(8)**

hint. Use logarithm of the function, and use and use the following power series

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots$$

4-4) Calculate $\displaystyle\lim_{x\to0}\left(\cos x\right)^{1/x\sin x}$ **(8)**

5. When f is defined as $f(x, y, z) = x^2 - xy + 3\sin z$, answer the following. (18 points)

5-1) Find the linearization $L(x,y,z)$ of $f(x, y, z)$ at the point $(x_0, y_0, z_0) = (2, 1, 0)$ **(8)**

5-2) Find an upper bound for the error incurred in replacing f by L
on the rectangle R: $|x - 2| \le 0.01$ $|y - 1| \le 0.02$ $|z| \le 0.01$. **(10)**

6. Answer the following questions. (14)

6-1) Show that $y\,dx + x\,dy + 4\,dz$ is exact. **(6)**

6-2) Evaluate the integral $\displaystyle\int_{(1,1,1)}^{(2,3,-1)} y\,dx + x\,dy + 4\,dz$ over the line segment from $(1,1,1)$ to $(2,3,-1)$. **(8)**