系所組別： 環境工程學系甲、乙組

考試科目： 工程數學

※ 考生請注意：本試題 □可 ☑不可 使用計算機

I. Please solve the following differential equations: (24 分, 每題 8 分)

A. $\dfrac{d^2y}{dx^2}+(\tan x)\dfrac{dy}{dx}=0$

B. $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+4y=\cos^2 x$

C. $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+13y=\delta(t-\pi)+\delta(t-3\pi)$ with $y(0)=1, y'(0)=1$

II. Please find the steady-state temperature distribution (26 分, 每題 13 分)

A. in a sphere: $\dfrac{\partial^2 u}{\partial r^2}+\dfrac{2}{r}\dfrac{\partial u}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2}+\dfrac{\cot\theta}{r^2}\dfrac{\partial u}{\partial \theta}=0$, for $0<r<c$ and $0<\theta<\pi$, with boundary condition $u(c,\theta)=\sin\theta,\ 0<\theta<\pi$

B. in a semi-infinite plate: $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$, for $0<x<\pi$ and $y>0$, with boundary conditions

$$\begin{cases} u(0,y)=0, \quad u(\pi,y)=e^{-y}, \quad y>0 \\ \left.\dfrac{\partial u}{\partial y}\right|_{y=0}=0, \quad 0<x<\pi \end{cases}$$

III. Please solve $\dfrac{\partial^2 u}{\partial t^2}=\dfrac{\partial^2 u}{\partial x^2}$, for $t>0$ and $0<x<1$, with initial conditions $\begin{cases} u(0,x)=0 \\ \left.\dfrac{\partial u}{\partial t}\right|_{t=0}=\sin \pi x \end{cases}$, $0<x<1$ and boundary conditions $\begin{cases} u(t,0)=0 \\ u(t,1)=0 \end{cases}$, $t>0$ (15 分)

IV. Compute $\oint_C (x^5+3y)dx+(2x-e^{y^3})dy$, where C is the circle $(x-1)^2+(y-5)^2=4$. (10 分)

V. In the Lotka-Volterra model, the populations of predator, x, and prey, y, are related as $\begin{cases} x'=-ax+bxy \\ y'=-cxy+dy \end{cases}$, where a, b, c, and d are positive constants. Please derive if there is any periodic solution for $\begin{cases} x'=0.004x(50-x-0.75y) \\ y'=0.001y(100-y-3.9x) \end{cases}$ and what is it, if yes. (15 分)

VI. Please derive the orders of accuracy for the second order of Runge-Kutta method used in single step and multiple steps for ordinary differential equation $\dfrac{dy}{dx}=f(x,y)$. (10 分)