

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please determine the limit for the following functions (if it exists). (15%)

$$(1) \lim_{x \rightarrow -2} \frac{x^2 - 4}{x^3 + 8}$$

$$(2) \lim_{\Delta x \rightarrow 0} \frac{\cos(\pi + \Delta x) + 1}{\Delta x}$$

$$(3) \lim_{x \rightarrow 1^+} (x-1)^{\ln x}$$

2. Please find the derivate of the following functions. (24%)

$$(1) f(x) = \log_5 \frac{4}{x^2 \sqrt{1-x}}$$

$$(2) f(x) = x(4^{-x})$$

$$(3) f(x) = \ln[x(x^2 - 2)^{2/3}]$$

$$(4) f(x) = 2x \sinh^{-1}(2x) - \sqrt{1+4x^2}$$

3. Please find or evaluate the integral of the following functions. (36%)

$$(1) \int \frac{x(x-2)}{(x-1)^3} dx$$

$$(2) \int_0^{\infty} x^2 e^{-x^2} dx$$

$$(3) \int \frac{e^{1/x}}{x^2} dx$$

$$(4) \int \frac{\sec^2 \theta}{\tan \theta (\tan \theta - 1)} d\theta$$

$$(5) \int \frac{1}{16+x^2} dx$$

$$(6) \int_0^{\pi/2} \frac{1}{1+(\tan x)^{\sqrt{3}}} dx$$

4. Please evaluate the iterated integral by converting to polar coordinates: (10%)

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \sin \sqrt{x^2 + y^2} dy dx.$$

5. Use cylindrical coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 4$ and above the upper nappe of the cone $z^2 = x^2 + y^2$. (hint: the solid is bounded above by surface $z = \sqrt{4 - x^2 - y^2}$ and below by surface $z = \sqrt{x^2 + y^2}$, respectively.) (15%)