

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

I. Please solve the following differential equations: (5 points for each one)

A.  $x \frac{dy}{dx} - (1+x)y = xy^2$

B.  $\frac{d^2y}{dx^2} - y = x + \sin x$

C.  $\frac{d^2y}{dx^2} + (\tan x) \frac{dy}{dx} = 0$

D.  $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = \ln x^2$

II. Please solve the system of differential equations 
$$\begin{cases} \frac{dx}{dt} + 2x + 6 \int_0^t y d\tau = -2 \\ \frac{dx}{dt} + \frac{dy}{dt} + y = 0 \end{cases}$$

with  $x(0) = -5$  and  $y(0) = 6$ . (10 points)

III. For a spherical empty tank with radius  $R$ , water is pumped into the tank at constant flow rate  $Q$ . If there is a circular hole of radius  $r$  at the bottom of the tank, please find the height of water level  $h$  as function of time  $t$ . (15 points)

IV. In finite difference method, three points  $y_{i+1}$ ,  $y_i$ , and  $y_{i-1}$  are used for  $y_i'$  and  $y_i''$ , please derive the computation schemes and the truncation error. (20 points)

V. For oil spill occurring on the top of a still pond, please find the concentration  $C$  of pollutant diffusing into the pond as function of time  $t$ . (20 points)

VI. Please solve the partial differential equations: (15 points)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \sin x \quad \text{with} \quad \begin{cases} u(x, 0) = 400 + \sin x, & 0 < x < \pi \\ t > 0, & u(0, t) = 400, \quad u(\pi, t) = 200 \end{cases}$$