※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please find the derivate of the following functions. (6 points each, 24 points total)
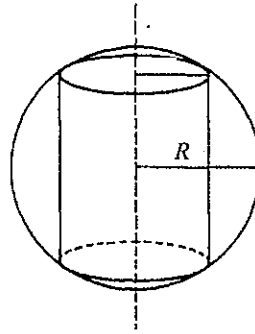
   (1) $f(x) = \dfrac{\cos x}{x^3}$

   (2) $f(x) = \ln \sqrt[3]{\dfrac{x-1}{x+1}}$

   (3) $F(x) = \displaystyle\int_0^{e^{2x}} \ln(t+1)dt$

   (4) $y = x\arctan 2x - \dfrac{1}{4}\ln(1+4x^2)$

2. Find the maximum volume of a right circular cylinder that can be inscribed in a sphere of radius R. (12 points)

3. A sector with central angle θ is cut from a circle of radius R, and the edges of the sector are brought together to form a cone. Find the magnitude of θ such that the volume of the cone is a maximum. (14 points)
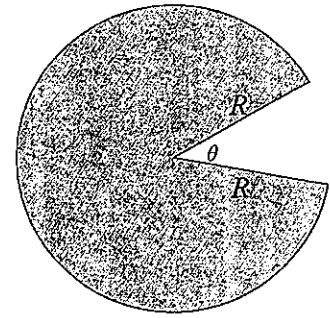

Figure for question 2.　　　Figure for question 3.

4. If $f(x) = \displaystyle\int_2^x \dfrac{dt}{\sqrt{1+t^4}}$, find $(f^{-1})'(0)$. (10 points)

5. Please find or evaluate the integral of the following functions. (6 points each, 30 points total)

   (1) $\displaystyle\int \left(2 - \tan\dfrac{\theta}{4}\right) d\theta$

   (2) $\displaystyle\int_{\ln 2}^{\ln 4} \dfrac{e^{-x}}{\sqrt{1-e^{-2x}}} dx$

   (3) $\displaystyle\int \dfrac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2} dx$

   (4) $\displaystyle\int \dfrac{2}{x\sqrt{1+4x^2}} dx$

   (5) $\displaystyle\int_0^2 e^{\sqrt{2x}} dx$

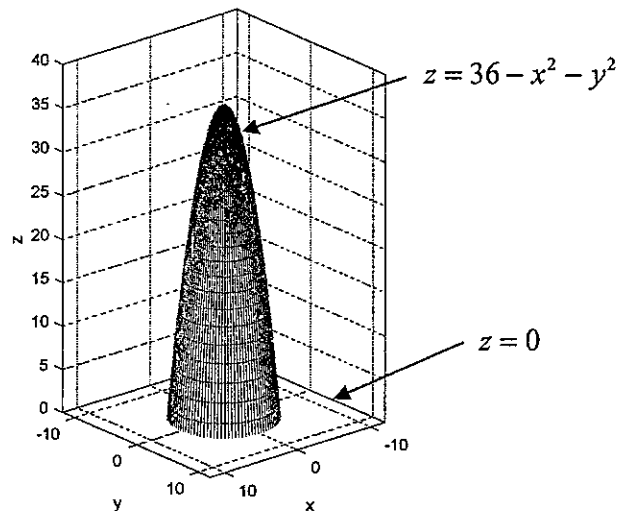6. Use a triple integral to find the volume of solid shown in the figure. (10 points)

$z = 36 - x^2 - y^2$

$z = 0$

Figure for question 6.