編號： 152　　國立成功大學 106 學年度碩士班招生考試試題

系　　所：環境工程學系

考試科目：工程數學　　考試日期：0213，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please solve the following equations: (5 points for each one)

A. $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$　　B. $\frac{d^2y}{dx^2}+y=\sec x$　　C. $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=0$

D. $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+13y=\delta(t-\pi)+\delta(t-3\pi)$ with $y(0)=1, y'(0)=1$

2. Please solve $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ with the following conditions: (10 points for each one)

A. $\begin{cases} u(x,0)=\sin x, \quad 0<x<\pi \\ t>0, \quad u(0,t)=5, \quad u(\pi,t)=5 \end{cases}$　　B. $\begin{cases} u(x,0)=100, \quad 0<x \\ t>0, \quad u(0,t)=25, \quad u(\infty,t)=100 \end{cases}$

C. $\begin{cases} u(x,0)=5, \quad 0<x<1 \\ t>0, \quad u(0,t)=1, \quad u(1,t)=2 \end{cases}$

3. Please solve the heat conduction equation in spherical coordinate as following (15 points)

$\frac{\partial u}{\partial t}=k\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)$, for $0<r<5$ and $t>0$ with $\begin{cases} u(r,0)=2, \quad 0<r<5 \\ t>0, \quad u(5,t)=1 \end{cases}$

4. For the first-order differential equation $\frac{dy}{dx}=-4y$ with $y(0)=2$, please derive the conditions of Δx to have positive and decreasing solution for modified Euler's method and second-order Runge-Kutta method, respectively. (20 points)

5. If $\rho(x,y)$ is the length density of a wire (mass per unit length), $m=\int_c \rho(x,y)ds$ is the mass of the wire. Find the mass of a wire having the shape of the semicircle $x=1+\cos t$, $y=\sin t$ and $0\le t\le\pi$, if the density at a point P is directly proportional to the distance from the y-axis. (15 points)