編號： 151

國立成功大學 107 學年度碩士班招生考試試題

系　　所：環境工程學系

考試科目：微積分

考試日期：0205，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

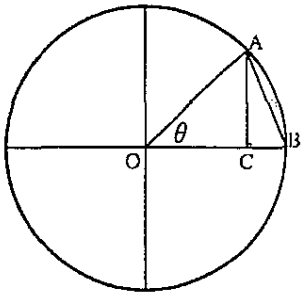
1. Answer the differentiation questions. (each 5 points, 20%)

   (1) $g(x)=\sqrt{x}+e^{x}\ln x$, find $g''(x)$.　(2) $x^2y-4x=5$, find $\frac{d^2y}{dx^2}$ implicitly in terms of x and y.

   (3) $y=(1+x)^{1/x}$, find $\frac{dy}{dx}$ using logarithmic differentiation.

   (4) $z=\frac{1}{2}(e^{x^2+y^2}-e^{-x^2-y^2})$, find the total differential dz.

2. As shown on the figure, let R be the ratio of the area of triangle ABC (abbreviated as ΔABC) to that of the shaded region formed by deleting ΔOAB from the circular sector subtended by angle θ. The segment AC is the height of ΔOAB. Please prove that the limit of R is 3 when θ approaches 0+. (10 points, 10%)



3. The oxygen deficit $D(x)$ of a river is a function of flow distance x from a pollution source with waste loading rate of L_0:

$$D=D_0e^{-\frac{k_a}{U}x}+\frac{k_dL_0}{k_a-k_r}(e^{-\frac{k_r}{U}x}-e^{-\frac{k_a}{U}x})$$

   Where by assuming that there is only one pollution source discharged at x=0, and L_0, D_0, U, k_a, k_d and k_r are constants. Find the critical distance of x that maximizes the oxygen deficit. (10 points, 10%)

4. Approximate $f(x)=x^2e^{-x}$ at $x=\frac{1}{3}$ using the 4th Maclaurin polynomial for $f(x)$. (10 points, 10%)

5. Answer the integration questions. (each 5 points, 30%)

   (1) $\int\frac{\ln\sqrt{x}}{x}dx$　(2) $\int\frac{3x^3+4x}{(x^2+1)^2}dx$　(3) $\int\frac{x}{\sqrt{1-x^4}}dx$

   (4) Prove $\int_a^b f(x)f'(x)dx=\frac{1}{2}\left([f(b)]^2-[f(a)]^2\right)$.　(5) $\int x^2\cos x dx$

   (6) $\int_0^\infty\int_0^\infty\frac{1}{(1+x^2+y^2)^2}dxdy$

6. Find the value of a such that the area bounded by $y=e^{-x}$, the x-axis, $x=-a$, and $x=a$ is $\frac{8}{3}$. (10 points, 10%)

7. Sketch and evaluate the solid region whose volume is given by the following iterated integral. (10 points, 10%)

$$\int_0^{2\pi}\int_0^{\pi}\int_2^5\rho^2\sin\phi d\rho d\phi d\theta$$