※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

I. Please solve the following equations: (5 points of each)

A. $x\dfrac{dy}{dx} + y = x^2 y^2$　　　　　B. $y'' - 5y' + 4y = \cos^2 x$

II. NO_x is defined as the sum of NO and NO_2 in the ambient atmosphere. NO is converted into NO_2 as first-order chemical reaction and the rate constant is 0.04 1/sec. If the NO/NO_x ratio is 0.9 for flue gas emitted from a boiler, how long will it take for the NO/NO_x ratio within this air parcel to become 0.4 ? (15 points)

III. If $\vec{F} = xy\vec{i} + y^2 z\vec{j} + z^3\vec{k}$, please evaluate $\iint_S (\vec{F}\cdot\vec{S})\,dS$, where S is the unit cube defined by $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$. (15 points)

IV. Please compute the temperature distribution within an object of various shapes and its initial temperature is 25°C: (15 points of each)

A. Cartesian coordinate $\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}$ with $\begin{cases} u(x,0) = 25, & 0 < x < 5 \\ t > 0, & u(0,t) = 5,\ u(5,t) = 5 \end{cases}$

B. Cylindrical bar $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r}$ with $\begin{cases} u(r,0) = 25, & 0 < r < 5 \\ t > 0, & u(5,t) = 5 \end{cases}$

C. Sphere $\dfrac{\partial u}{\partial t} = k\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial u}{\partial r}\right)$ with $\begin{cases} u(r,0) = 25, & 0 < r < 5 \\ t > 0, & u(5,t) = 5 \end{cases}$

V. Please derive the conditions for stable solution for the explicit and implicit finite difference methods used to solve the partial differential equation $\dfrac{\partial T}{\partial t} = k\dfrac{\partial^2 T}{\partial x^2}$. (15 points)