編號： 141 **國立成功大學109學年度碩士班招生考試試題**

系　　所：環境工程學系

考試科目：工程數學

考試日期：0210，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please find the solutions for the following differential equations:(5 points for each one)

A. $y''+(y')^2+1=0$　　B. $x^2y''-4xy'+6y=\ln x^2$　　C. $y''-5y'+4y=\cos^2 x$

D. $y''+4y'+13y=\delta(t-\pi)+\delta(t-3\pi)$ with $y(0)=1, y'(0)=1$　　E. $y''+2y'+y=xe^{-x}$

2. Water is pumped into an empty cylindrical tank of diameter D at a constant flow rate Q. A round hole of diameter d is at the bottom of the tank and water flows out from the hole by gravity. Please find the height of water level in the tank as a function of time. (15 points)

3. Find the solutions of $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ for the following conditions.(10 points for each one)

A. $\begin{cases} u(x,0)=1, & 0<x<1 \\ t>0, \quad u(0,t)=0, & \left.\frac{\partial u}{\partial x}\right|_{x=1}=-u(1,t) \end{cases}$　　B. $\begin{cases} u(x,0)=1, & x>0 \\ t>0, \quad u(0,t)=5, & u(\infty,t)=1 \end{cases}$

4. Find the solutions of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for the following conditions. (15 points for each one)

A. $\begin{cases} u(0,y)=1, \quad \lim_{x\to\infty} u(x,y)=0, & 0<y<1 \\ \left.\frac{\partial u}{\partial y}\right|_{y=0}=0, \left.\frac{\partial u}{\partial y}\right|_{y=1}=-u(x,1), & x>0 \end{cases}$　　B. $\begin{cases} u(0,y)=0, \quad u(\pi,y)=e^{-y}, & y>0 \\ \left.\frac{\partial u}{\partial y}\right|_{y=0}=0, & 0<x<\pi \end{cases}$

5. If the finite difference equation $\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}+p(x_i)\frac{y_{i+1}-y_{i-1}}{2h}+q(x_i)y_i=r(x_i)$ is used for the differential equation $y''+p(x)y'+q(x)y=r(x)$, please derive the truncation error. (10 points)