國立成功大學

110學年度碩士班招生考試試題

編　號：145

系　所：環境工程學系

科　目：微積分

日　期：0202

節　次：第 3 節

備　註：不可使用計算機

編號： 145

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Verify that the y-coordinate of the inflection point of the following function is $S/2$. (10 points, 10%)

$$y=\frac{S}{1+\alpha e^{-x/\beta}},\quad \alpha>0,\quad \beta>0,\quad S>0$$

2. Find k such that $\int_0^\infty\int_0^\infty f(x,y)dydx=1$, where $f(x,y)=\begin{cases} ke^{-(x^2+y^2)}, & x\ge 0,\quad y\ge 0 \\ 0 \quad , & \text{elsewhere}\end{cases}$. (10 points, 10%)

3. Find the sum of the following convergent series by using a well-known function (e.g., a natural logarithmic function, an exponential function, a trigonometric function, etc.). (Total 12 points, 12%)

(1) $\sum_{n=0}^{\infty}\frac{2^n}{3^n n!}$　　(2) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{5^n n}$

4. A single COVID-19-infected individual enters a community of n susceptible individuals at time 0. Let x be the number of newly COVID-19-infected individuals in the community at time t. Assuming that the COVID-19 spreads at a rate proportional to the product of the total number infected, $1+x$, and the number not infected, $n-x$, so $dx/dt=k(1+x)(n-x)$, where k is a rate constant. Solve for x as a function of t. (12 points, 12%)

5. Answer the integration questions. (Total 36 points, 36%)

(1) $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}e^{-x^2/2}dx$　　(2) $\int_0^\infty\frac{e^{-1/x}}{x^2}dx$　　(3) $\int_1^2(-x+2)\sin\left(\frac{n\pi x}{2}\right)dx$

(4) $\int\frac{1}{x\sqrt{9x^2-49}}dx$　　(5) $\int x\sin 3x^2dx$　　(6) $\int_{-2}^{2}\int_0^{4-y^2}\int_0^x dzdxdy$

6. As shown in Figure 1, find the distance BC such that $0\le BC\le 3$ and yields the absolute maximum θ. (10 points, 10%)

7. As shown in Figure 2, a circle of radius a is centered at (0, a). Find the area of the shaded region as a function of h ($0\le h\le 2a$). (10 points, 10%)

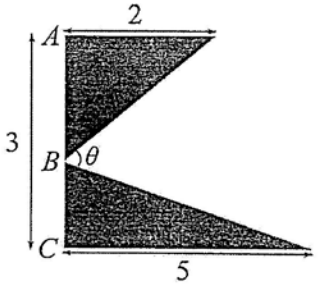

Figure 1

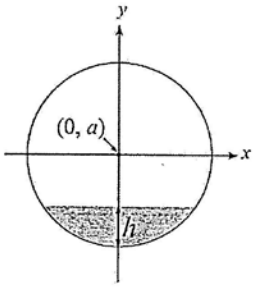

Figure 2