

國立成功大學
110學年度碩士班招生考試試題

編 號：146

系 所：環境工程學系

科 目：工程數學

日 期：0202

節 次：第 3 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please solve the following differential equations: (5 points for each one)

- A. $y''+5y'+6y=3e^{-2x}+e^{3x}$ B. $y''+4y'+5y=\delta(t-2\pi)$ with $y(0)=0, y'(0)=0$
 C. $y'''+y''=e^x \cos x$ D. $xy''+y'=x$ E. $xy'+y=x^2y^2$

2. Please solve the following partial differential equations: (10 points for each)

A. $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \sin x$ with $\begin{cases} u(x, 0) = 400 + \sin x, & 0 < x < \pi \\ t > 0, & u(0, t) = 400, \quad u(\pi, t) = 200 \end{cases}$

B. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r}$, with $\begin{cases} u(r, 0) = 2, \quad \left. \frac{\partial u}{\partial t} \right|_{t=0} = 1, & 0 < r < 1 \\ t > 0, \quad \left. \frac{\partial u}{\partial r} \right|_{r=1} = 0 \end{cases}$

3. For the first-order differential equation $\frac{dy}{dx} = f(x, y)$ with $y(0) = y_0$, please derive the truncation error and the overall order of accuracy for second-order Runge-Kutta method. (20 points)

4. The Dufort-Frankel method for the partial differential equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ is

$$\frac{T_i^{n+1} - T_i^{n-1}}{2\Delta t} = \frac{T_{i+1}^n - (T_i^{n+1} + T_i^{n-1}) + T_{i-1}^n}{\Delta x^2},$$

please derive the conditions for consistency. (10 points)

5. A spherical particle of mass m_p is injected horizontally into a still air at velocity u_0 . If the drag force is proportional to the particle velocity as $F_d = -f u_p$, where u_p is the particle velocity, please find the maximum horizontal traveling distance of the particle. (10 points)

6. In the mathematical analysis of the organic waste and oxygen contents in a natural stream, Streeter and Phelps (1925) has made the following assumptions: plug flow, first-order reaction of organic waste with respect to its own concentration, unit stoichiometric ratio of oxygen consumption with respect to organic waste, and the interfacial transfer flux of oxygen into the stream as $k(C\alpha^* - C\alpha)$. Please derive the concentrations of organic waste and oxygen as functions of distance from discharge if the concentrations of organic waste and oxygen are C_{bo} and C_{ao} , respectively, at $x = 0$. (15 points)