國立成功大學
111學年度碩士班招生考試試題

編　號：146

系　所：環境工程學系

科　目：微積分

日　期：0219

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please find the derivative of the following functions.(4 points each,12points total)

   (1) $f(x)=5x(4^{-3x})$　　(2) $g(x)=\log_5\dfrac{4}{x^2\sqrt{1-x}}$　　(3) $h(t)=\sin(\arccos t)$

2. Find an equation of the tangent line to the graph of the function, $1+\ln xy=e^{x-y}$, at the point (1, 1). (6 points)

3. A sector with central angle θ is cut from a circle of radius R (see Figure 1), and the edges of the sector are brought together to form a cone. Find the magnitude of θ such that the volume of the cone is a maximum. (15 points)

4. A hole is cut through the center of a sphere of radius r (see Figure 2). The height of the remaining spherical ring is h. Find the volume of the ring. (12 points)

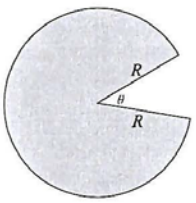

Figure 1

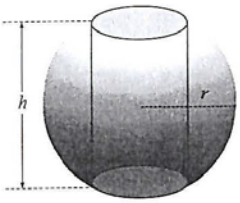

Figure 2

5. Let $a>0$ and $b>0$. Show that the area of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is πab (see Figure 3). (6 points)

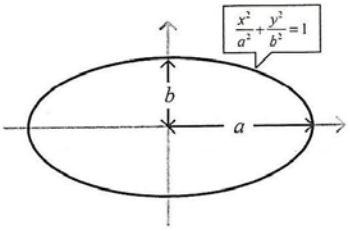

Figure 3

6. Solve the differential equation. (6 points each, 24 points total)

   (1) $\dfrac{dy}{dx}=\dfrac{10x^2}{\sqrt{1+x^3}}$　　(2) $e^x y'+4e^x y=1$, where $y'=\dfrac{dy}{dx}$　　(3) $(x^2+y^2)dx-2xydy=0$

   (4) $\dfrac{dy}{dt}=\dfrac{3y}{20}-\dfrac{y^2}{1600}$, $y(0)=15$

7. Please find or evaluate the integral of the following functions. (5 points each, 25 points total)

   (1) $\int_0^{\pi}\tan\dfrac{\theta}{3}d\theta$　　(2) $\int_{-2}^{0}x^2e^{x^3/2}dx$　　(3) $\int\dfrac{1}{(x^2+5)^{3/2}}dx$

   (4) $\int x^5\ln x dx$　　(5) $\int_0^2\dfrac{dx}{x^2-2x+2}$