國立成功大學
111學年度碩士班招生考試試題

編　號：147

系　所：環境工程學系

科　目：工程數學

日　期：0219

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1.Please solve the following equations, where $y'=\frac{dy}{dx}$ and $y''=\frac{d^2y}{dx^2}$: (5 points for each one)

A.$(6xy-4y^2)y'+(2x+3y^2)=0$　　B. $y''+2y'=2x+5-e^{-2x}$

C. $y''-2y'+y=\cos^2 x$　　D. $x^2y''-xy'+y=x^3$

2.Please solve the following equations: (15 points for each)

A. $\frac{\partial^2 u}{\partial t^2}=k\frac{\partial^2 u}{\partial x^2}$ with $\begin{cases} u(x,0)=0, & \left.\frac{\partial u}{\partial t}\right|_{t=0}=\sin(\pi x), \quad 0<x<1 \\ t>0, & u(0,t)=0, \quad u(1,t)=0 \end{cases}$

B. $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial u}{\partial \theta}=0$ with $\begin{cases} 0<r<a, & 0<\theta<2\pi \\ u(a,\theta)=sin3\theta \end{cases}$

3.For vector $\vec{F}=xy\vec{i}+y^2z\vec{j}+z^3\vec{k}$, evaluate the surface integral $\iint_s (\vec{F}\cdot\vec{n})dS$, where $\vec{n}$ is the normal unit vector to surface *S* and *S* is the unit cube defined by $0\le x\le 1$, $0\le y\le 1$, and $0\le z\le 1$. (10 points)

4.A object of mass *m* lying on a flat surface is attached to the end of a spring whose constant for Hooke's law is *k*. If the object is moved to the right at distance *L* from the equilibrium position and then released, please derive the differential equation for the motion of the object and solve its position as function of time for the following conditions: (10 points for each one)

A.The surface is frictionless;

B.The friction coefficient is *f*;

C.The surface is frictionless and a force *Fi* is applied to the object instantly at one second after releasing.

5.For first-order differential equation $\frac{dy}{dx}=f(x,y)$, the computation schemes for 4th-order Runge-Kutta method are $y_{i+1}=y_i+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$, where *h* is the interval of *x* and

$$k_1=\mathrm{h}*\mathrm{f}(x_i,y_i),$$

$$k_2=\mathrm{h}*\mathrm{f}(x_i+\tfrac{h}{2},y_i+\tfrac{k_1}{2}),$$

$$k_3=\mathrm{h}*\mathrm{f}(x_i+\tfrac{h}{2},y_i+\tfrac{k_2}{2}),\text{ and}$$

$$k_4=\mathrm{h}*\mathrm{f}(x_i+h,y_i+k_3).$$

Please show the computation schemes for equation $\frac{d^2y}{dx^2}-3x^2y\frac{dy}{dx}-2x^3y^2=4x$. (10 points)