國立成功大學

114學年度碩士班招生考試試題

編　號：110

系　所：環境工程學系

科　目：工程數學

日　期：0210

節　次：第 3 節

注　意： 1. 不可使用計算機
2. 請於答案卷(卡)作答，於試題上作答，不予計分。

I. Please solve the following differential equations: (6 points for each one)

A. $y''' - 5y'' + 6y' = 8 + 2\sin x$

B. $y'' - 2y' + y = e^x \tan x$

C. $x^2 y'' - 4xy' + 6y = \ln x^2$

D. $y'' + 4y' + 5y = \delta(t - 2\pi)$ with $y(0) = 0, y'(0) = 0$

E. $\dfrac{dx}{dy} = -\dfrac{4y^2 + 6xy}{3y^2 + 2x}$

II. Please solve $\iint_S xz^2 \, dS$, where S is the portion of the cylinder $y = 2x^2 + 1$ in the first octant bounded by $x = 0$, $x = 2$, $z = 4$, and $z = 8$. (10 points)

III. Please solve $\dfrac{\partial u}{\partial t} = k \dfrac{\partial^2 u}{\partial x^2}$ with the following conditions: (15 points for each one)

A. $\begin{cases} u(x,0) = 1, \quad 0 < x < 1 \\ t > 0, \quad u(0,t) = 0, \quad \left.\dfrac{\partial u}{\partial x}\right|_{x=1} = -u(1,t) \end{cases}$

B. $t > 0, u(x,0) = \begin{cases} 0, & x < 0 \\ e^{-x}, & x > 0 \end{cases}$

IV. Please derive the condition for stable solution if the explicit finite difference method,
$\dfrac{T_i^{n+1} - 2T_i^n + T_i^{n-1}}{\Delta t^2} = k \dfrac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$, is used to solve the partial differential equation $\dfrac{\partial^2 T}{\partial t^2} = k \dfrac{\partial^2 T}{\partial x^2}$. (15 points)

V. A spherical particle of mass m_p is injected horizontally into a still air with initial velocity U_0. The drag force on the particle, F_d, is proportional to particle velocity with constant coefficient k, that is, $F_d = -kU$, where U is the particle velocity. Please find the maximum horizontal traveling distance of the particle. (15 points)