國立成功大學

115學年度碩士班招生考試試題

編　號：104

系　所：環境工程學系

科　目：工程數學

日　期：0203

節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. Please solve the following differential equations: (5 points for each one)

A. $2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4x = cos^2 x$ B. $\frac{d^2y}{dx^2} + y = sec\, x$

C. $\frac{d^2y}{dx^2} - y = x + \sin x$

D. $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = \delta(t-\pi) + \delta(t-3\pi)$ with $y(0) = 1$ and $\left.\frac{dy}{dt}\right|_{t=0} = 1$

2. For a cylindrical empty tank of diameter D, water is pumped into the tank at constant flow rate of Q. If there is a circular hole of diameter d at the bottom of the tank, please find the height of water level, h, as a function of time, t. (15 points)

3. Please write down the computation equations explicitly if the fourth order Runge-Kutta method is used for the differential equation $y'' - 2y' + 3xy^2 = x^2$ with initial conditions $y(0) = 1$ and $y'(0) = 0$. (15 points)

4. Please solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with $\begin{cases} u(0,y) = 1, \quad \lim_{x\to\infty} u(x,y) = 0, \quad 0 < y < 1 \\ \left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, \left.\frac{\partial u}{\partial y}\right|_{y=1} = -u(x,1), \quad x > 0 \end{cases}$. (15 points)

5. Please solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ with $\begin{cases} u(x,0) = 0, \quad x > 0 \\ t > 0, \quad u(0,t) = 1, \quad u(\infty,t) = 0 \end{cases}$. (15 points)

6. For n set of experimental data (x_i, y_i) where i is from 1 to n, least-square method is used to determine the coefficient a and intercept b for linear regression equation $y = ax + b$. Please derive the equations for coefficient a and intercept b. (20 points)