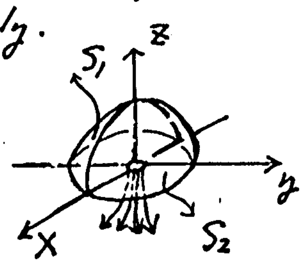
1) A tank, surrounded by two surfaces S_1 & S_2, is <u>initially filled</u> with water. At the bottom of the tank, there is a hole of <u>unit area</u>, through which the water drains with <u>the velocity</u> $\sqrt{z}$, where z is the height of the water. Find the height of the water in the tank at any time and the time to drain the water completely.
(20%)



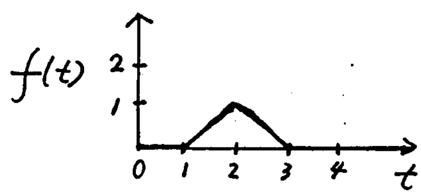
$S_1: x^2+y^2=1-z,$

$S_2: x^2+y^2<1$

2) Solve the ordinary differential equation

$$y''-4y'+3y=f(t), \quad t\geq 0$$

with initial condition $y(0)=y'(0)=0$. Also, $f(t)$ is defined as
(15%)



3) Solve the partial differential equation

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

with boundary condition

$x=1, \quad u=0$

$x=2, \quad u=0$

and initial condition

$t=0, \quad u=1.$ (20%)

4) In problem (1), find

(a) the unit outward normal vector of surface S_1 (5%)

(b) A vector defined as $\vec{u}=x^2\vec{i}+y\vec{j}-z\vec{k}$, where $\vec{i},\vec{j},\vec{k}$ are the unit vectors in x, y & z directions. Find the integral $\iint_{S=S_1+S_2}\vec{u}\cdot\vec{n}\,dS$, where $\vec{n}$ is the unit outward normal vector of S, and S is the surfaces of S_1 & S_2. (10%)

5). For the function defined in the complex plane

$$f(z) = \frac{1 - z + 6z^2}{(z^2 + \frac{1}{4})(z-2)}$$

where z is the complex variable.
Find the integral $\int_C f(z)\,dz$, along the closed path C in the countercurrent sense, where C is the circle of radius 1 about the point (0.5, 0.5) in the complex plane (10%)

6) Solve the coupled ordinary differential equations (20%)

$$\begin{cases} \dfrac{d^2 y_1}{dx^2} = -3y_1 + 2(y_2 - y_1) \\ \dfrac{d^2 y_2}{dx^2} = -2(y_2 - y_1) \end{cases}$$

with

$$\begin{cases} x = 0, & y_1 = 1, \quad y_2 = 2 \\ x = 0, & \dfrac{dy_1}{dx} = 0, \quad \dfrac{dy_2}{dx} = 0 \end{cases}$$