1. A reactor having only five rectangular sides is required to have a given volume V. Determine its dimensions so that the surface area will be a minimun. (10%)

2. The absorption of light in a very thin transparent layer is proportional to the thickness of the layer and to the amount incident on that layer. Formulate this in terms of a differential equation and solve it. (10%)

3. Show that

$$\int_{-\infty}^{\infty} \frac{\cos sx}{k^2+x^2}\,dx = \frac{\pi}{k}e^{-ks}, \quad s>0, k>0,$$ by means of the residue theorem. (10%)

4. The temperature distribution in a long thin bar of constant cross section and homogeneous material, which is oriented along the x-axis and is perfectly insulated laterally, is governed by the one-dimensional heat equation

$$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}, \quad c^2 = K/\sigma\rho$$

where K is the thermal conductivity, σ is the specific heat, and ρ is the density of the material. If the ends x=0 and x=ℓ of the bar are kept at temperature zero, and f(x) is the initial temperature in the bar. Determine the temperature u(x,t). (20%)

5. By means of line integral, find the area of the interior of the cardioid $r=a(1-\cos\theta)$, $0\le\theta\le 2\pi$. (10%)

6. Find the volume of the tetrahedron that has the following vertices (0,1,2), (5,5,6), (1,2,1), (3,3,1) (10%)

7. Show how can you reduce the nonlinear differential equation $P(x)y' + Q(x)y = R(x)y^a$ where a is any real number, to a linear one. (5%)

8. (a) Solve the problem

$y'' - 3y' + 2y = 4x + 3e^{3x}$, $y(0)=3$, $y'(0)=3$ by

(1) one of the standard methods; (5%)

(2) the Laplace transform method. (5%)

(b) What are the advantages of the Laplace transform method over other standard method for obtaining the particular solution of a nonhomogeneous differential equation? (5%)

9. (a) Let $g_1(x), g_2(x), \ldots$ be any orthogonal set of functions on an interval $a\le x\le b$ and let f(x) be a given function which can be represented in terms of the g_j's by a convergent series

$f(x) = \sum_{n=1}^{\infty} c_n g_n(x)$.

Show how can you determine the constants c_n. (5%)

(b) Which orthogonal set of functions yields Fourier series? Derive the Euler formulas. (5%)