1. 某生解 $\int_0^\infty e^{-2t}\sin 3t\,dt$ 解法如下:

<sol> $\mathcal{L}\{\sin 3t\} = \int_0^\infty e^{-st}\sin 3t\,dt = \frac{3}{s^2+9}$

令 $s=2$ 得 $\int_0^\infty e^{-2t}\sin 3t\,dt = \frac{3}{13}$

如上法又解 $\int_0^\infty e^{2t}\sin 3t\,dt$

<sol> 令 $s=-2$ 得 $\int_0^\infty e^{2t}\sin 3t\,dt = \frac{3}{13}$

結果得如下之結論 $\int_0^\infty e^{-2t}\sin 3t\,dt = \int_0^\infty e^{2t}\sin 3t\,dt$

你認為对不对? 請說明理由 (7%)

2. 試求 $\int_0^\infty e^{-t}\frac{\sin t}{t}\,dt$ 之值 (8%)

3. 試解下列微分方程式

① $e^y y' = x + e^y - 1$ $\quad (y' = \frac{dy}{dx})$ (5%)

② $xy' + 2y = 2e^{x^2}$ (5%)

③ $(D+1)(D^2+4)y = \sin 2x + x$ $\quad (Dy = \frac{dy}{dx},\ D^2y = \frac{d^2y}{dx^2})$ (5%)

④ 联立 $\begin{cases} (D-2)x + 2Dy = e^{2t} \\ (2D-3)x + (3D-1)y = 0 \end{cases}$ $\quad (Dx = \frac{dx}{dt},\ Dy = \frac{dy}{dt})$ (8%)

4. (i) 試將 $f(x) = \frac{x^2}{4}$ 以 Fourier Series 展開 (8%)

即 $f(x) = a_0 + \sum_{n=1}^{\infty} a_n\cos nx + b_n\sin nx$

(ii) 試利用(i)求 $\sum_{k=1}^{\infty}\frac{1}{k^2}$ 之值. (7%)

5. 試求 $x^2+y^2+z^2=5$ 球面在點 $P(1,0,2)$ 之单位法向量. (7%)

6. 試解

$$\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial x^2} + \frac{2}{x}\frac{\partial C}{\partial x} \qquad (10\%)$$

$t=0 \quad C = f(x)$

$x=R \quad C = C_1$

$x=0 \quad C$ 為有限值

7. 已知 1st kind Bessel function 為

$$J_\nu(x) = x^\nu \sum_{m=0}^{\infty} \frac{(-1)^m \, x^{2m}}{2^{2m+\nu} \, m! \, \Gamma(\nu+m+1)} \qquad \Gamma(x) \text{ 為 Gamma function}$$

試証 $J_{-n}(x) = (-1)^n J_n(x)$ (7%)

8. Bessel Equation $x^2 y'' + x y' + (x^2 - \nu^2) y = 0$ 之解為 $y = A J_\nu(x) + B J_{-\nu}(x)$ 但當 $\nu \in N$ 時, $J_\nu(x)$ 與 $J_{-\nu}(x)$ 相依（見題7）,因此定義第二類型 Bessel function

即 $Y_\nu = \frac{1}{\sin \nu\pi} \left[J_\nu(x) \cos \nu\pi - J_{-\nu}(x) \right]$

$$Y_n(x) = \lim_{\nu \to n} Y_\nu(x)$$

① 試寫出 $Y_n(x)$ (5%)

② 証明 $Y_n(x)$ 為 Bessel Equation 之解 (5%)

9. 已知 Green's Theorem $\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\,dy = \oint_C F_1 dx + F_2 dy$

① 試証一封閉曲間所圍的面積為

$$A = \frac{1}{2} \oint_C x\,dy - y\,dx \qquad (7\%)$$

② 試求橢圓 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 之面積. (6%)