

1. Please compute the results of the following questions (5% for each one):

a. $\frac{d}{dx} \left[\left(\frac{x}{x^2+1} \right)^e \right]$

b. $\int (1+e)^x dx$

c. $\int \frac{\ln x}{x} dx$

d. $\int e^x \sin x dx$

2.(5%) If f is a differentiable function such that $y = f(x)$ satisfies the equation: $x^2 + x^2 y^2 - x - 2y + 1 = 0$, find dy/dx .

3.(10%) Find a triangle of maximum area inscribed in a circle of radius r and compute its area.

4.(15%) If $f''(x)$ exists on $[a, b]$ and

$$\int_a^b f(x) dx = \frac{h}{2} [f_0 + 2*f_1 + 2*f_2 + \dots + 2*f_{n-1} + f_n] + E_n$$

where $f_0, f_1, f_2, \dots, f_n$ are the values of $f(x)$ at $x_0, x_0+h, x_0+2*h, \dots, x_0+n*h$, respectively. Show that the error E_n is $-(b-a)^3 * f''(c) / (12 * n^2)$ where c is some point between a and b .

5.(10%) Find the derivative and its domain for $f(x) = (|x| - |x-1|)^2$.

6.(10%) Find the volume of the solid under the surface $z = x^2 + y^2$, above the xy -plane, and inside the cylinder $x^2 + y^2 = 2y$.

7.(10%) Find the value of $e^{1/3}$ to an accuracy of 3 decimal places.

8.(10%) If $f(x, y, z) = x e^{yz} + yz e^x$. Calculate $\partial^3 f / \partial x^2 y$

9.(10%) Find the area of the region bounded by $y = x \ln(x)$ and the x -axis.