1. Please find the general solutions for the following equations: (5% for each one)
   a. $x^2y''-xy'+y = lnx$
   b. $y''+y = 4x+10sinx$
   c. $xy'+(1+x)y = exp(-x)$
   d. $(1+x^2)y'+2xylny = 0$
   e. $(3x+y-2)y'-(2x+2y+1) = 0$

2. Please solve the following partial differential equation:(10%)

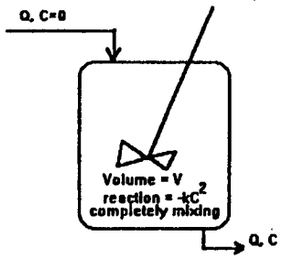
$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0 \quad with \quad u(0,y)=u_o, \frac{\partial u}{\partial x}\Big|_{x=a}=0; u(x,0)=0, u(x,b)=f(x)$$

3. For function f(x) = 0 at -1 < x < 0 and f(x) = 1 at 0 < x < 1,
   a. What is its Fourier series expansion ?
   b. What are the values of the Fourier series computed in (a) at x = -0.5, 0, and 0.5, respectively, for the summation to be infinite number of terms ? (10%)

4. The temperature distribution in a solid plate is given as $T(x,y)=5+2x^2+y^2$. Please determine the direction of heat conduction at point (2,4). (10%)

5. For the numerical integration of function f(x) from x=a to x=b, what is the Simpson's 1/3 method ? Please also estimate its error term and express the result in terms of a, b, the grid spacing, and the derivative of function f(x). (15%)

6. Waste water with concentration for pollutant of C is treated in a continuous stirred tank reactor (CSTR), where the concentration is uniformly distributed within the reactor, with volume of V, the input and output volume flow rate of Q. The CSTR is shown in the following figure. In the CSTR, the pollutant reacts according to $-kC^2$. Please find the time required to reduce the effluent concentration to 1/4 of the initial value with and without the effects of chemical reaction. (15%)



7. A chemical spill occurs in a still lake and the chemical is transported downward by diffusion. Please compute the vertical concentration distribution of the chemical within the lake as a function of time and distance from the surface. Assume that there is no chemical reaction, the concentration of the chemical on the surface is constant and that the lake is so deep that the change in concentration is still far from the bottom of lake. (15%)