一、(30%) 請完成下列各子題之微分與積分運算：

a. 微分運算：

1. $\log_{10}(x^2+2)$　　2. $\dfrac{\log_{10} x}{x}$　　3. $\left(\dfrac{x}{x^2+1}\right)^e$

4. $\tanh^{-1} x$　　5. 2^{2x}

b. 積分運算：

1. $\int_{-1}^{1} (1+e)^x dx$　　2. $\int \cosh^3 x dx$　　3. $\int_0^1 \int_0^{2\sqrt{1-y^2}} (4-x^2-4y^2) dx dy$

4. $\int \sqrt{x}(2x+1)^2 dx$　　5. $\int \sqrt{e^{2x}+e^x+1}\, dx$

二、(25%) 請求出 $F(x)=x^2 e^{-x}$ 之極大、極小點及反曲點，並在同一座標平面內繪出 F, F' 及 F'' 之圖形？

三、(20%) 試完成與空氣污染擴散有關之積分：

$$\int_0^{+\infty}\int_0^{+\infty} e^{-(x^2+y^2)}\, dy\, dx$$

四、(25%) 環境地質學家去野外採四種岩石樣本，設 X_1, X_2, X_3 為第Ⅰ, Ⅱ, Ⅲ 類岩石佔總樣本重之百分比，則第Ⅳ類之百分比為 $1-X_1-X_2-X_3$，因此只有 X_1, X_2 及 X_3 為獨立事件，其聯合機率密度函數可設為：

$$f(x_1,x_2,x_3)=\begin{cases} k\, x_1 x_2 (1-x_3) & 0\le x_1\le 1,\ 0\le x_2\le 1,\ 0\le x_3\le 1,\ 0\le x_1+x_2+x_3\le 1 \\ 0 & \text{otherwise} \end{cases}$$

則 k 可由 $1=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1,x_2,x_3)\, dx_3\, dx_2\, dx_1$ 來決定，請完成此問題（求出 k 值）

1. Please find the general solutions for the following equations: (5% for each one)

   a. $x^2y''-xy'+y = \ln x$

   b. $y''+y = 4x+10\sin x$

   c. $xy'+(1+x)y = \exp(-x)$

   d. $(1+x^2)y'+2xy\ln y = 0$

   e. $(3x+y-2)y'-(2x+2y+1) = 0$

2. Please solve the following partial differential equation:(10%)

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0 \quad with \quad u(0,y)=u_o, \frac{\partial u}{\partial x}\Big|_{x=a}=0; u(x,0)=0, u(x,b)=f(x)$$

3. For function $f(x) = 0$ at $-1 < x < 0$ and $f(x) = 1$ at $0 < x < 1$,

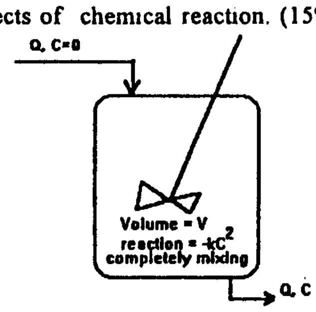
   a. What is its Fourier series expansion ?

   b. What are the values of the Fourier series computed in (a) at x = -0.5, 0, and 0.5, respectively, for the summation to be infinite number of terms ? (10%)

4. The temperature distribution in a solid plate is given as $T(x,y)=5+2x^2+y^2$. Please determine the direction of heat conduction at point (2,4). (10%)

5. For the numerical integration of function f(x) from x=a to x=b, what is the Simpson's 1/3 method ? Please also estimate its error term and express the result in terms of a, b, the grid spacing, and the derivative of function f(x). (15%)

6. Waste water with concentration for pollutant of C is treated in a continuous stirred tank reactor (CSTR), where the concentration is uniformly distributed within the reactor, with volume of V, the input and output volume flow rate of Q. The CSTR is shown in the following figure. In the CSTR, the pollutant reacts according to $-kC^2$. Please find the time required to reduce the effluent concentration to 1/4 of the initial value with and without the effects of chemical reaction. (15%)



7. A chemical spill occurs in a still lake and the chemical is transported downward by diffusion. Please compute the vertical concentration distribution of the chemical within the lake as a function of time and distance from the surface. Assume that there is no chemical reaction, the concentration of the chemical on the surface is constant and that the lake is so deep that the change in concentration is still far from the bottom of lake. (15%)

Thermodynamics

Problem 1 (20%)

Describe briefly the following energy systems: (a) the combined cycle (b) fuel cell, (c) batteries, and (d) the cogeneration.

Problem 2 (20%)

The power required to operate a steady-state compressor is 3.56 Kw. Air enters the compressor at 1 bar and 300 K at a rate of 1 kg/min and leaves at 7 bars and 500 K. Determine (a) the rate of heat transfer, in kJ/h, (b) the entropy change of the air, in Kj/(min K), and (c) the entropy change of the environment which receives the heat transferred at 288 K, in kJ/(min K). (d) Is the process reversible, irreversible, or impossible?

problem 3 (20%)

At what temperature, will CO be 10% of the total moles of products if CO is burned with the stoichiometric amount of O_2 at 2-atm pressure?

Problem 4 (20%)

(a) Derive the expression $c_p - c_v = -T(\partial v/\partial T)_p^2(\partial P/\partial v)_T$. (b) At 500 K the values of v, β, and K_T for solid copper are 7.115 mL/gmol, 54.2×10^{-6} K^{-1}, and 0.837×10^{-7} cm^2/N, respectively. Determine the value of $c_p - c_v$ in J/gmol C. ($\beta = (1/v)(\partial v/\partial T)_p$, $K_T = -(1/v)(\partial v/\partial P)_T$).

Problem 5 (20%)

A Carnot heat engine receive 90 kJ from a reservoir at 900 K. It rejects heat to the environment at 300 K. One-fifth of its work output is used to derive a Carnot refrigerator. The refrigerator rejects 60 kJ to the environment at 300 K. Find (a) the work output of the heat engine, (b) the efficiency of the heat engine, (c) the temperature of the low-temperature reservoir for the refrigerator, and (d) the coefficient of performance (COP) of the refrigerator.

...ysical constants and conversion factors

Physical constants

...gadro's number	$N_A = 6.023 \times 10^{26}$ atoms/kgmol
...versal gas constant	$R_u = 0.08205$ L·atm/(gmol·K)
	= 8.314 kJ/(kgmol·K)
	= 0.08314 bar·m³/(kgmol·K)
	= 8.314 kPa·m³/(kgmol·K)
...ck's constant	$h = 6.626 \times 10^{-34}$ J·s/molecule
...mann's constant	$k = 1.380 \times 10^{-23}$ J/(K·molecule)
...d of light	$c = 2.988 \times 10^{10}$ cm/s
...dard gravity	$g = 9.80665$ m/s²

Conversion factors

...th	1 cm = 0.3937 in = 10^4 μm = 10^8 Å
	1 km = 0.6215 mi = 3281 ft
	1 kg = 2.205 lb_m
	1 N = 1 kg·m/s² = 0.2248 lb_f
...ure	1 bar = 10^5 N/m² = 0.9869 atm
	= 100 kPa
	1 torr = 1 mmHg at 0°C = 1.333 mbar
	= 1.933×10^{-2} psi
	1 mbar = 0.402 inH_2O
...ne	1 L = 0.0353 ft³ = 0.2642 gal = 61.025 in³ = 10^{-3} m³
...ty	1 g/cm³ = 1 kg/L = 62.4 lb_m/ft³ = 10^3 kg/m³
...y	1 J = 1 N·m = 1 V·C
	= 0.7375 ft·lb_f = 10 bar·cm³ = 0.624×10^{19} eV
	1 kJ = 0.948 Btu = 737.6 ft·lb_f = 10^{-2} bar·m³
	1 kJ/kg = 0.431 Btu/lb
	1 W = 1 J/s = 3.413 Btu/h
	1 kW = 1.3405 hp = 737.3 ft·lb_f/s
...ity	1 m/s = 2.237 mi/h = 3.60 km/h = 3.281 ft/s
...ic heat	1 kJ/(kg·K) = 0.2389 Btu/(lb_m·°F)
...erature	$T(K) = \frac{5}{9}[T(°F) + 459.67] = T(°C) + 273.15 = T(°R)/1.8$

...rived units and common multipliers

1. Some SI derived units

...al quantity	Unit	Symbol	Definition
	newton	N	1 kg·m/s²
...ne	pascal	Pa	1 kg/m·s² (= 1 N/m²)
	bar	bar	10^5 kg/m·s² (= 10^5 N/m²)
...y	joule	J	1 kg·m²/s² (= 1 N·m)
	watt	W	1 kg·m²/s³ (= 1 J/s)
...c quantity	coulomb	C	1 A·s
...c potential ...ence	volt	V	1 kg·m²/(A·s³)(= 1 A·Ω)
...c resistance	ohm	Ω	1 kg·m²/(A²·s³) (= 1 V/A)
...c capacitance	farad	F	1 A²·s⁴/(kg·m²) (= 1 C/V)

2. Names and symbols for common multipliers of SI units

...ier	Prefix	Symbol
	giga	G
	mega	M
	kilo	k
	deci	d
	centi	c
	milli	m
	micro	μ
	nano	n

Logarithms to the base 10 of the equilibrium constant K_p

$$K_p = \frac{(p_E)^{\nu_E}(p_F)^{\nu_F}}{(p_A)^{\nu_A}(p_B)^{\nu_B}} \text{ for the reaction } \nu_A A + \nu_B B \rightleftharpoons \nu_E E + \nu_F F$$

Numbered reactions:

(1) $H_2 \rightleftharpoons 2H$
(2) $O_2 \rightleftharpoons 2O$
(3) $N_2 \rightleftharpoons 2N$
(4) $\frac{1}{2}O_2 + \frac{1}{2}N_2 \rightleftharpoons NO$
(5) $H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2$
(6) $H_2O \rightleftharpoons OH + \frac{1}{2}H_2$
(7) $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2$
(8) $CO_2 + H_2 \rightleftharpoons CO + H_2O$

Temp., K	log K_p values for reactions numbered above (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
298	−71.224	−81.208	−159.600	−15.171	−40.048	−46.054	−45.066	−5.018
500	−40.316	−45.880	−92.672	−8.783	−22.886	−26.130	−25.025	−2.139
1000	−17.292	−19.614	−43.056	−4.062	−10.062	−11.280	−10.221	−0.159
1200	−13.414	−15.208	−34.754	−3.275	−7.899	−8.789	−7.764	+0.135
1400	−10.630	−12.054	−28.812	−2.712	−6.347	−7.003	−6.014	+0.333
1600	−8.532	−9.684	−24.350	−2.290	−5.180	−5.662	−4.706	+0.474
1700	−7.666	−8.706	−22.512	−2.116	−4.699	−5.109	−4.169	+0.530
1800	−6.896	−7.836	−20.874	−1.962	−4.270	−4.617	−3.693	+0.577
1900	−6.204	−7.058	−19.410	−1.823	−3.886	−4.177	−3.267	+0.619
2000	−5.580	−6.356	−18.092	−1.699	−3.540	−3.780	−2.884	+0.656
2100	−5.016	−5.720	−16.898	−1.586	−3.227	−3.422	−2.539	+0.688
2200	−4.502	−5.142	−15.810	−1.484	−2.942	−3.095	−2.226	+0.716
2300	−4.032	−4.614	−14.818	−1.391	−2.682	−2.798	−1.940	+0.742
2400	−3.600	−4.130	−13.908	−1.305	−2.443	−2.525	−1.679	+0.764
2500	−3.202	−3.684	−13.070	−1.227	−2.224	−2.274	−1.440	+0.784
2600	−2.836	−3.272	−12.298	−1.154	−2.021	−2.042	−1.219	+0.802
2700	−2.494	−2.892	−11.580	−1.087	−1.833	−1.828	−1.015	+0.818
2800	−2.178	−2.536	−10.914	−1.025	−1.658	−1.628	−0.825	+0.833
2900	−1.882	−2.206	−10.294	−0.967	−1.495	−1.442	−0.649	+0.846
3000	−1.606	−1.898	−9.716	−0.913	−1.343	−1.269	−0.485	+0.858
3100	−1.348	−1.610	−9.174	−0.863	−1.201	−1.107	−0.332	+0.869
3200	−1.106	−1.340	−8.664	−0.815	−1.067	−0.955	−0.189	+0.878
3300	−0.878	−1.086	−8.186	−0.771	−0.942	−0.813	−0.054	+0.888
3400	−0.664	−0.846	−7.736	−0.729	−0.824	−0.679	+0.071	+0.895
3500	−0.462	−0.620	−7.312	−0.690	−0.712	−0.552	+0.190	+0.902

Source: Based on data from the JANAF Tables, NSRDS-NBS-37, 1971, and revisions published in *Journal of Physical and Chemical Data* through 1982.

Ideal-gas properties of air

T, K; h, kJ/kg; u, kJ/kg; s°, kJ/(kg·K)

T	h	p_r	u	v_r	s°	T	h	p_r	u	v_r	s°
200	199.97	0.3363	142.56	1707.	1.29559	460	462.02	6.245	329.97	211.4	2.13407
210	209.97	0.3987	149.69	1512.	1.34444	470	472.24	6.742	337.32	200.1	2.15604
220	219.97	0.4690	156.82	1346.	1.39105	480	482.49	7.268	344.70	189.5	2.17760
230	230.02	0.5477	164.00	1205.	1.43557	490	492.74	7.824	352.08	179.7	2.19876
240	240.02	0.6355	171.13	1084.	1.47824	500	503.02	8.411	359.49	170.6	2.21952
250	250.05	0.7329	178.28	979.	1.51917	510	513.32	9.031	366.92	162.1	2.23993
260	260.09	0.8405	185.45	887.8	1.55848	520	523.63	9.684	374.36	154.1	2.25997
270	270.11	0.9590	192.60	808.0	1.59634	530	533.98	10.37	381.84	146.7	2.27967
280	280.13	1.0889	199.75	738.0	1.63279	540	544.35	11.10	389.34	139.7	2.29906
285	285.14	1.1584	203.33	706.1	1.65055	550	555.74	11.86	396.86	133.1	2.31809
290	290.16	1.2311	206.91	676.1	1.66802	560	565.17	12.66	404.42	127.0	2.33685
295	295.17	1.3068	210.49	647.9	1.68515	570	575.59	13.50	411.97	121.2	2.35531
300	300.19	1.3860	214.07	621.2	1.70203	580	586.04	14.38	419.55	115.7	2.37348
305	305.22	1.4686	217.67	596.0	1.71865	590	596.52	15.31	427.15	110.6	2.39140
310	310.24	1.5546	221.25	572.3	1.73498	600	607.02	16.28	434.78	105.8	2.40902
315	315.27	1.6442	224.85	549.8	1.75106	610	617.53	17.30	442.42	101.2	2.42644
320	320.29	1.7375	228.42	528.6	1.76690	620	628.07	18.36	450.09	96.92	2.44356
325	325.31	1.8345	232.02	508.4	1.78249	630	638.63	19.84	457.78	92.84	2.46048
330	330.34	1.9352	235.61	489.4	1.79783	640	649.22	20.64	465.50	88.99	2.47716
340	340.42	2.149	242.82	454.1	1.82790	650	659.84	21.86	473.25	85.34	2.49364
350	350.49	2.379	250.02	422.2	1.85708	660	670.47	23.13	481.01	81.89	2.50985
360	360.58	2.626	257.24	393.4	1.88543	670	681.14	24.46	488.81	78.61	2.52589
370	370.67	2.892	264.46	367.2	1.91313	680	691.82	25.85	496.62	75.50	2.54175
380	380.77	3.176	271.69	343.4	1.94001	690	702.52	27.29	504.45	72.56	2.55731
390	390.88	3.481	278.93	321.5	1.96633	700	713.27	28.80	512.33	69.76	2.57277
400	400.98	3.806	286.16	301.6	1.99194	710	724.04	30.38	520.23	67.07	2.58810
410	411.12	4.153	293.43	283.3	2.01699	720	734.82	32.02	528.14	64.53	2.60319
420	421.26	4.522	300.69	266.6	2.04142	730	745.62	33.72	536.07	62.13	2.61803
430	431.43	4.915	307.99	251.1	2.06533	740	756.44	35.50	544.02	59.82	2.63280
440	441.61	5.332	315.30	236.8	2.08870	750	767.29	37.35	551.99	57.63	2.64737
450	451.80	5.775	322.62	223.6	2.11161	760	778.18	39.27	560.01	55.54	2.66176

一、作文：50%

請以下列文字為基礎，申論你心目中的現代女性美與男性美之典型

在社會美中，以人的美為中心。美的行為、美的品德、美的情操、美的理想、美的實踐、美的生活等等，都集中地體現在人的身上。人的美，包括外在美和內在美。

外在美，也叫儀表美。是指人的形體、服裝、髮式、言談、舉止、姿態、表情、神態等構成的美；內在美，是指人的心靈和精神的美，它包括人的思想、感情、理想、智慧、品德、情操等因素。一般來說，內在美與外在美不是對立的，而是互為表裡，相得益彰的。人的內在美是通過外在美顯露出來的，內在的心靈和精神美要表現在外在形象上；一個人的衣著打扮、言談舉止、表情神態，在一定程度上表現著他的思想、感情、品德、理想、智慧、情操等內在美。在這種統一中，內在美起主導、決定作用；或者說，人的內在美決定著人的衣著打扮、言談舉止、表情神態。一個有遠大理想、道德高尚的人，一個脫離了低級趣味的人，必然會舉止端莊、言談謙和、衣著樸素整潔；必然是與人為善，公而忘私，助人為樂，自覺地遵守社會道德；必然是謙虛謹慎，艱苦樸素，嚴格要求自己。

在我們看到內在美與外在美統一的時候，也要看到兩者在某些人身上還有不一致的地方。有的人長相漂亮，但靈魂骯髒，「金玉其外，敗絮其中」；有的人穿著華麗，但知識淺薄，低級庸俗，「繡花枕頭，一包草」；有的人雖然貌不出眾，但有「內秀」，有一顆高尚、純潔、閃閃發光的心靈。因此，人們在現實生活中，在長期的審美活動中，逐漸認識到內在美更重於外在美；總是注意在長期的社會實踐中考查人，把重心落到內在美上。俗語說：「鳥美美在毛，人美美在心」。我們對鳥的美，更多的偏重於形式；對於人的美，更多地偏重於內在美。人的價值就在於有內心世界的美。因此，對整個人的美起決定作用的，是人的心靈，而不是儀表。在人類的文化史上有很多藝術家，例如貝多芬，托爾斯泰等，都不是什麼美男子，但他們的思想、感情、品德、智慧、情操，卻閃爍著美的光輝！

因此，容貌體形之美固然是一種寶貴的天賦，但是在人這種高級動物身上，它不能決定人的本質。外貌美易逝，這種美是暫時的；內在美常存，給人的美是長期的。內在美是人們衡量一個人美不美的最後尺度。電影《巴黎聖母院》，塑造了敲鐘人卡西摩多，他相貌醜陋，人們一開始看到他時，心裡有一種不大愉快的感覺，但是，隨著故事情節的展開，他的高尚心靈慢慢地顯露出來，他有一顆善良的心，有一種真的愛，因而，人們越來越同情他，熱愛他，讚美他。

I. Translate TWO of the following passages into CHINESE: 10%

1. The principal thesis of positivism is that there is a precise criterion by which all significant discourse may be accurately distinguished from meaningless discourse, and that according to this criterion most of what has passed for philosophy in the past is strictly senseless.

2. When things have the name in common and the definition of being which corresponds to the name is the same, they are called synonymous. Thus, for example, both a man and an ox are animals. Each of these is called, by a common name, an animal, and the definition of being is also the same; for if one is to give the definition of each - what being an animal is for each of them - one will give the same definition.

3. All men by nature desire to have knowledge. An indication of this is the delight that we take in the senses; quite apart from the use that we make of them, we take delight in them for their own sake, and more than of any other this is true of the sense of sight.

II. Translate ONE of the following passages into ENGLISH: 10%

1. 為什麼生命的誕生會帶來歡樂，而生命的終結卻與哀傷脫離不了關係呢？

2. 在人生的過程中，教育應俱有啟發以及引導受教育者，通往幸福人生的功能．

III. Composition: 30%
Choose ONE of the topics listed below and write an organized essay.

1. What is your opinion about the relationship between aesthetics and ethics? Be sure to support your explanation with specific reasons.

2. What is your opinion about the relationship between nature and culture? Be sure to support your explanation with specific reasons.