1. In a plug flow reactor with velocity u and length L, species A is input at concentration C_0 and decays a reaction of order n, $r = -kC^n$, please compute the concentration of A at the exit。 (10分)
   If the effect of Brownian diffusion at axial direction is included, what is the concentration, assuming the diffusivity of D and $n=1$. (10分)

2. A particle of diameter d_p and density ρ_p is injected into still air at an initial velocity u_0. (a) If the spherical particle is injected horizonally, what is the horizontal travelling distance before the particle stops. (b) If the particle is injected upward, what is the maximum upward travelling distance. In both cases, consider gravity and the drag force as ku where u is velocity of particle. (20分)

3. Please solve P.D.E. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with $\begin{cases} u(0,y)=0, \quad u(\pi,y)=e^{-y} \text{ at } y>0 \\ \text{and } \left.\frac{\partial u}{\partial y}\right|_{y=0} = 0, \text{ at } 0<x<\pi. \end{cases}$
   (15分)

4. For the unsteady heat conduction equation $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$ with

   ① $\begin{cases} T(0,x)=0 \\ T(t,0)=T_1 \\ T(t,\infty)=T_1 \end{cases}$ ② $\begin{cases} T(0,x)=T_1 \\ T(t,0)=0 \\ T(t,\infty)=T_2 \end{cases}$ ③ $\begin{cases} T(0,x)=0 \\ T(t,0)=T_1 \\ T(t,h)=T_1 \end{cases}$ ④ $\begin{cases} T(0,x)=0 \\ T(t,0)=T_1 \\ T(t,h)=0 \end{cases}$ ⑤ $\begin{cases} T(0,x)=T_1 \\ T(t,0)=0 \\ T(t,h)=0 \end{cases}$ ⑥ $\begin{cases} T(0,x)=T_0 \\ T(t,0)=T_1 \\ T_x(t,h)=0 \end{cases}$

   ⑦ $\begin{cases} T(0,x)=T_0 \\ T(t,0)=0 \\ T_x(t,h)=0 \end{cases}$ ⑧ $\begin{cases} T(0,x)=T_0 \\ T_x(t,0)=0 \\ T(t,h)=T_2 \end{cases}$ ⑨ $\begin{cases} T(0,x)=T_0 \\ T_x(t,0)=0 \\ T_x(t,h)=0 \end{cases}$ ⑩ $\begin{cases} T(0,x)=T_0 \\ T_x(t,0)=T_1 \\ T_x(t,h)=T_1 \end{cases}$ ⑪ $\begin{cases} T(0,x)=T_0 \\ T_x(t,0)=0 \\ T_x(t,h)=T_1 \end{cases}$ where $T_x = \frac{\partial T}{\partial x}$

   A. Which can be solved by letting $T(t,x) = \theta(t)\,\Xi(x)$ and using separation method directly?
   B. Which has non-zero constant steady state solution, i.e. $T = \text{constant} \neq 0$ as $t \to \infty$.
   (答對一個4分,答錯倒扣4分).

5. 是非題. (每小題答對2分,答錯倒扣2分).
   A. Bessel functions為 orthogonal set.
   B. $P_n(x)$ is Legendre polynomial, 則 $P_n(\infty)=0$
   C. $P_n(x)$ is Legendre polynomial, $P_{2k+1}(0)=0$, where $k \in N$.
   D. In numerical method with finite difference method, numerical solution converges to the true solution as $\Delta x \to 0$
   E. Function $f(t)=e^{t^2}$, the Laplace transform of $f(t)$ exists.

6. Please use two different methods to solve $(2xy+2y^2)dx + (x^2+4xy)dy = 0$. (15分)
   Write down details of the computation steps.