1. A spherical water droplet with diameter dp is falling by gravity in a still air column. Please drive the concentration of NH_3 in the water droplet as a function of distance from the top of the column. Assume the concentration of NH_3 within the water droplet at the top of the column is zero and that at the surface of droplet is at a constant value of Co. Note that NH_3 diffuses into the droplet with a diffusivity of D and neglect the effects of hydrolysis and chemical reaction for simplicity. (20分)

2. Please drive the error term for the Simpson 1/3 method for the integration of $\int_a^b f(x)dx$ if (b-a) is divided into n equi-spaced intervals. (15分)

3. For the differential equation $y'' + \lambda y = 0$ with the following conditions: (a) $y(0) = y(1) = 0$; (b) $y'(0) = y'(1) = 0$. Please find the values of λ and the corresponding eigenfunctions for y to have non-trivial solutions. (10分)

4. For a 2-D Laplace's equation $\nabla^2 u = 0$ with the following boundary conditions.

(1). Which can be solved by letting $u(x,y) = X(x)Y(y)$ (separation of variable method) directly ?
(答對五個3分,答錯倒扣3分)。

(2). Which can be solved by separation of variables with some proper transformation, i.e., $u(x,y) = X(x)Y(y) + \Phi(x)$ or $\Phi(y)$. Write down how you transform the variable and the answer of $\Phi(x)$ or $\Phi(y)$.
(五個答對5分,答錯倒扣5分)。

(a) $u(0,y)=0,\ u_x(a,y)=0,\ u(x,0)=f(x),\ u_y(x,b)=0$;
(b) $u_x(0,y)=f(y),\ u(a,y)=0,\ u(x,0)=0,\ u(x,b)=0$;
(c) $u(0,y)=u_o,\ u_x(a,y)=0,\ u(x,0)=u_1,\ u_y(x,b)=u_o$;
(d) $u_x(0,y)=u(0,y),\ u_x(a,y)=u(a,y),\ u(x,0)=0, u_y(x,b)=u_o$;
(e) $u_x(0,y)=u(0,y),\ u(a,y)=0,\ u_y(x,0)=u_1,\ u(x,b)=u_1$;
(f) $u_x(0,y)=0,\ u_x(a,y)=u_o,\ u(x,0)=u_1,\ u_y(x,b)=0$;
(g) $u_x(0,y)=u_o,\ u(a,y)=u_o,\ u_y(x,0)=u_1,\ u(x,b)=0$,
(h) $u(0,y)=u_o,\ u_x(a,y)=u_1,\ u_y(x,0)=0,\ u_y(x,b)=0$;
(i) $u(0,y)=u_o,\ u_x(a,y)=u_o,\ u(x,0)=u_1,\ u_y(x,\infty)=u_1$;
(j) $u_x(0,y)=0,\ u(a,y)=u_o,\ u_y(x,0)=u_1,\ u(x,b)=u_1$;
(k) $u(0,y)=u_o,\ u(\infty,y)=0,\ u(x,0)=u_1,\ u(x,b)=u_1$;
(l) $u_x(0,y)=0,\ u_x(a,y)=u_o,\ u_y(x,0)=u_1, u_y(x,b)=u_o$

Where u_o and u_1 are constant values and $u_x = \frac{\partial u}{\partial x}$; $u_y = \frac{\partial u}{\partial y}$.

5. Find the solutions for the following equations. (各題5分)

(a) $y'' + 2y' + y = e^{-x}\ln x$

(b) $x^2y'' - 2xy' + 2y = x^3\ln x$

(c) $\frac{dy}{dx} + 6y + 9\int_0^x y(\tau)d\tau = 1$