1. Please derive the condition for stable solution if the explicit finite difference method is used to solve the partial differential equation $\frac{\partial C}{\partial t} = \alpha \frac{\partial^2 C}{\partial x^2}$. (17%)

2. Please use similarity method to solve the partial differential equation (15%)

$$\frac{\partial C}{\partial t} = \alpha \frac{\partial^2 C}{\partial x^2} \text{ with } t=0,\ C(0,x)=C_0,\quad 0<x<\infty$$
$$t>0,\ C(t,0)=C_1 \text{ and } C(t,\infty)=C_0 .$$

3. The buoyancy force on a floating object is $\mathbf{B} = -\iint_S p\, \mathbf{n}\, dS$, where p is the fluid pressure. The pressure p is related to the density of the fluid $\rho(x,y,z)$ by a law of hydrostatics: $p = \nabla\rho(x,y,z)\mathbf{g}$, where $\mathbf{g}$ is the constant acceleration of gravity. If the weight of the object is $\mathbf{W} = m\mathbf{g}$, show what the $(\mathbf{B} + \mathbf{W})$ is ? (15%)

4. In the predator-prey model of Volterra, we are concerned with only two species: the predators and their prey. The populations of prey increase according to the Malthusian law with coefficient a and the loss is proportional to the number of encounters between the two species with coefficient b. The predators die away as a first order decay with coefficient c and increase according to the access of food with coefficient d. Determine the relationship between the populations of predators and prey at any time. (16%)

5. Find the solutions for the following questions.
   A. $(x-1)^2 y'' + 2(x-1) y' - 4 y = 0$. (5%)
   B. eigenvalues and eigenvectors for matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 0 & 1 \end{bmatrix}$. (6%)
   C. $y' - x \cdot y'' - (y'')^3 = 1$ (5%)

6. 是非題（每小題答對3分，答錯倒扣3分，未作答零分）。
   A. The Laplace transform for function $f(t) = exp(-t^2)$ does not exist.
   B. The two functions $f_1(x) = x$ and $f_2(x) = x^2$ are linearly independent.
   C. For Legendre polynomial $Pn(x)$, $Pn(\infty) = 0$.
   D. Both of the first and the second kind of Bessel functions, $Jn(x)$ and $Yn(x)$, are orthogonal sets.
   E. The Fourier series for function $f(x) = \begin{cases} 1 & -1 \le x < 0 \\ 0 & 0 \le x < 1 \end{cases}$ converge to 1, 0, and 0 for x at −0.5, 0, and 0.5, respectively.
   F. For any scalar function f, curl(grad f) = 0.
   G. If $\mathbf{a}$ is a constant vector and $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, $\mathbf{a} \times (\nabla \times \mathbf{r}) = 0$.