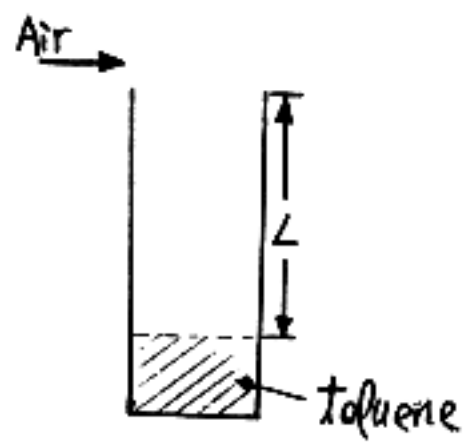
1. Please solve the following ordinary differential equations:(5% for each one)

A. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 1 = \cos^2 x$

B. $\frac{d^2y}{dx^2} + y = \sec x$

2. In order to estimate the fugitive emission of toluene from open storage tank as shown on the right, diffusion through stagnant air at steady state is assumed. Please compute the emission flux of toluene. Note that the system is kept at constant temperature and pressure and the airborne concentration of toluene at the interface is Cs. (15%)



3. A spherical particle of diameter d_p is injected into still air at initial velocity u_o and the drag force is proportional to the particle velocity.
(a) If the particle is injected horizontally, what is the maximum horizontal distance that the particle can travel ?
(b) If the particle is injected upward, what is the maximum upward traveling distance ? (20%)

4. Please solve the partial differential equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ with initial and boundary conditions as
(a) u(x,0) = 5, for 0<x<1; and at t >0, u(0,t) = 0, u(1,t) = 5;
(b) u(x,0) = 5, for x >0; and at t>0, u(0,t) = 0, u(∞,t) = 5. (20%)

5. Two different numerical methods, Euler's method and second-order Runge-Kutta method, are used to solve the differential equation: $\frac{dy}{dx} = -\alpha y$, where α is greater than zero. Please derive the conditions for the interval h to achieve stable solution. (20%)

6. The first order derivative f_i' is approximated by three points f_i, f_{i+1}, and f_{i+2}, which are equally spaced. Please derive the formula and its order of accuracy. (15%)