1. Please find the solutions y for the following differential equations: (10%)

   (A). $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=x^4e^x$

   (B). $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+13y=\delta(t-\pi)+\delta(t-3\pi)$ with $y(0)=1$ and $y'(0)=0$.

2. Please find the values of k and the corresponding non-trivial solutions of y for the ordinary differential equation $y''+ky=0$ with the following boundary conditions: (A) $y(0)=0$ and $y(\pi)=0$; (B) $y'(0)=0$ and $y'(\pi)=0$. (10%)

3. Finite difference method is used to solve the boundary-value problem $y''-2xy'+3x^2y=e^{-x}$ with $y(0)=1$ and $y'(1)=0$, please write down the finite difference equations for the governing equation at point x_i and the boundary points. (20%)

4. Please drive the computation schemes and their corresponding orders of accuracy for the first-order derivative f'_i with the following three equival-spaced points: (A) f_{i-1}, f_i, and f_{i+1}; (B) f_i, f_{i+1}, and f_{i+2}. (20%)

5. Please solve the partial differential equation $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$ with the following boundary conditions:

   (A) $0<x<1$, $0<y<\pi$ and $u(0,y)=0$, $u(1,y)=0$, where $0<y<\pi$; $u(x,0)=1$, $u(x,\pi)=2$, where $0<x<1$;

   (B) $0<x<\pi$, $y>0$ and $u(0,y)=0$, $u(\pi,y)=e^{-y}$, where $y>0$; $\frac{\partial u(x,0)}{\partial y}=0$, where $0<x<\pi$. (20%)

6. In the mathematical analysis of the organic waste and oxygen contents in a natural stream as shown below, Streeter and Phelps (1925) had made the following assumptions: plug flow, first-order reaction of organic waste with respect to its own concentration, unit stoichiometric ratio of oxygen consumption with respect to organic waste, and the interfacical transfer flux of oxygen as $k(C_a^*-C_a)$. Please derive the concentrations of organic waste and oxygen as functions of distance if their concentrations are C_{bo} and C_{ao}, respectively, at discharging point, that is, $x=0$. (20%)