

本科總分共104分.

1. For partial differential equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ with the following conditions

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|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| a. $T(0,x)=0$ | b. $T(0,x)=T_0$ | c. $T(0,x)=T_0$ | d. $T(0,x)=T_0$ | e. $T(0,x)=T_0$ | f. $T(0,x)=T_0$ |
| $T(t,0)=T_1$ | $T(t,0)=0$ | $T(t,0)=T_1$ | $T_x(t,0)=T_1$ | $T_x(t,0)=0$ | $T(t,0)=T_1$ |
| $T(t,\infty)=T_1$ | $T(t,l)=0$ | $T_x(t,l)=0$ | $T(t,l)=0$ | $T(t,l)=0$ | $T(t,l)=T_1$ |
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|-----------------|-----------------|---------------|---------------|-------------------|-----------------|
| g. $T(0,x)=T_0$ | h. $T(0,x)=T_0$ | i. $T(0,x)=0$ | j. $T(0,x)=0$ | k. $T(0,x)=T_0$ | l. $T(0,x)=T_0$ |
| $T(t,0)=0$ | $T(t,0)=0$ | $T(t,0)=T_1$ | $T(t,0)=T_1$ | $T_x(t,0)=T(x,0)$ | $T(t,0)=0$ |
| $T_x(t,l)=T_2$ | $T_x(t,l)=0$ | $T_x(t,l)=0$ | $T(t,l)=T_2$ | $T_x(t,l)=0$ | $T(T,\infty)=0$ |

A. Which can be solved by using $T(t,x)=H(t)R(x)$ (separation of variables) directly? (12分)

B. Which has non-zero constant steady state solution and write down the steady state solution. (12分)

(答對每個4分, 答錯倒扣4分)

2. Find the eigenvalues and eigenfunctions of $y'' + ky = 0$ with

- (16分) (A) $y'(0)=0, y'(1)=0$; (B) $y(0)+y'(0)=0, y(1)=0$.

3. Please derive the condition for stable solution if the explicit finite difference method is used to solve

the partial differential equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$. (20分)

4. The buoyancy force on a floating object is $\mathbf{B} = \int p \mathbf{n} dS$, where p is the fluid pressure. The pressure p is related to the density of the fluid $\rho(x,y,z)$ by a law of hydrostatics: $p = -\int \rho(x,y,z) \mathbf{g}$, where \mathbf{g} is the constant acceleration of gravity. If the weight of the object is $\mathbf{W} = m\mathbf{g}$, show what the $(\mathbf{B} + \mathbf{W})$ is?

(20分)

5. Please use similarity method to solve the partial differential equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ with the following

conditions: $T(0,x) = T_0$, for $x > 0$; and

for $t > 0$; $T(t,0) = T_1, T(t,\infty) = T_0$. (20分)