1. Please solve the following differential equations. (5 points for each one)

A. $y'' + (y')^2 + 1 = 0$ B. $y'' + y = \sec x$ C. $y'' - 5y' + 4y = \cos^2 x$ D. $y''' + y'' = e^x \cos x$

2. Please solve the following system of differential equations. (10 points for each one)

A. $x'' + 2x' - x + y' - 3y = \sin t$ $\quad x' + 4x + y' - 2y = e^{-t}$

B. $y' + 2y + 6\int_0^t z(\tau)d\tau = -2$ with $y(0) = -5$ $\quad y' + z' + z = 0$ $\quad z(0) = 6$

3. The following reactions occur in a smog chamber:

$$NO_2 \xrightarrow{h\nu} NO + O$$
$$O + O_2 \longrightarrow O_3$$
$$O_3 + NO \longrightarrow O_2 + NO_2$$

and the rate constants are J, k2, and k3, respectively. If the initial concentrations for NO, NO_2, and O_3 are $[NO]_o$, $[NO_2]_o$, and $[O_3]_o$, respectively. Please compute the concentrations of NO, NO_2, and O_3 at any time. Note that pseudo-steady state approximation is in the computation. That is the formation and consumption rates of oxygen atom are assumed to be the same. (20 points)

4. Please derive the order of accuracy for using multi-step second order Runge Kutta method to evaluate the initial boundary value differential equation $\frac{dy}{dx} = f(x, y)$ with $y(a) = y_o$ for $y(x=b)$. Note that the interval from a to b is assumed to be divided into n equi-spaced steps. (20 points)

5. Gaussian plume is generally assumed in the computation of pollutant dispersion from stack. Please derive the equation of Gaussian plume dispersion for a stack with height of H and the wind speed is u, the emission rate of pollutant is q, and the dispersion coefficient is D. For simplicity, the dispersion is assumed to be in the vertical direction only. Hint: the Fourier integral transform of $\exp(-x^2/4p^2)$ is $2\sqrt{\pi}\, p \exp(-p^2\alpha^2)$. (20 points)