編號：311 308 系所：環境工程學系甲組，乙組 科目：工程數學

1. For a cylindrical tank with diameter of D, water is pumped into the tank at a flow rate of Q. If there is a circular hole of diameter d at the bottom of the tank and the height of water level is ho at t = 0, please find the height of water level as a function of time. (15 points)

2. The Lokta-Voltera predictor-prey model shows that the growth rates for populations of predictor x and prey y are as the followings, where a, b, c, and d are constant parameters. Please derive the relationship between the populations of predictor x and prey y at any time. (15 points)

$$\frac{dx}{dt} = -ax + bxy$$

$$\frac{dy}{dt} = dy - cxy$$

3. Let P(x,y) and Q(x,y) be continuous and have continuous first partial derivatives in a simply connected region of the xy-plane. If $\int_A^B Pdx + Qdy$ is independent of the path, please derive the value of $\oint_C Pdx + Qdy$ along a smooth simple close curve C in the region. (20 points)

4. Finite difference method is used to solve the boundary-value ordinary differential equation $\frac{d^2y}{dx^2} + 4x^2 y\frac{dy}{dx} - 3y^2 = 4x$ with boundary conditions $y(x=0)=0$ and $y(x=1)=1$. If four intervals are used in the computation and the finite difference equations are expressed as $AX = B$ where X is the unknown column matrix $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$, where y_1, y_2, y_3 and y_4 are the unknown dependent variable at $x = 0.2, 0.4, 0.6$ and 0.8. Please derive the matrices A and B. (20 points)

5. Please solve the partial differential equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ with the initial condition as $u(0,x)=1$ for $0 < x < 5$, and the boundary conditions are $u(t,0)=0$ and $u(t,5)=1$ for $t > 0$. (15 points)

6 Dry adiabatic lapse rate, which is the vertical gradient of temperature $\frac{dT}{dz}$, is an important parameter in determining the atmospheric stability for pollutant dispersion. The dry adiabatic lapse rate is derived

(背面仍有題目,請繼續作答)

from the idea gas law equation $PV = nRT$, the hydraulic static equation $\frac{dP}{dz} = -\rho(z)g$, and the adiabatic expansion relationship for air $\frac{T(z_2)}{T(z_1)} = \left[\frac{P(z_2)}{P(z_1)}\right]^{(\gamma-1)/\gamma}$. Please derive the dry adiabatic lapse rate $\frac{dT}{dz}$. (15oints)