

1. (25 pt) Please find the derivatives of the following functions:

$$(a) \frac{1}{x - \sin x} \quad (b) (4-x^{2/5})^{-5/2} \quad (c) \sqrt{2 + \cos^2 x} \quad (d) \tan\theta - \theta \sec^2\theta \quad (e) \frac{\sqrt{1+t^2} - 1}{\sqrt{1+t^2} + 1}$$

2. (5 pt) Please evaluate the limit of $\lim_{x \rightarrow \pi/6} \frac{\cos(2x) - (1/2)}{x - \pi/6}$

3. (10 pt) The isothermal compressibility of a gas is the relative rate of change of the volume V with respect to the pressure P , at a constant temperature T , that is, $\frac{1}{V} \frac{dV}{dP}$. For a sample of an ideal gas, the temperature, pressure, and volume satisfy the equation $PV = kT$, where k is a constant related to the number of molecules of gas present in the sample. Show that the isothermal compressibility of such a gas is the negative reciprocal of the pressure:

$$\frac{1}{V} \frac{dV}{dP} = -\frac{1}{P}$$

4. (15 pt) A single cubical salt crystal is growing in a beaker of salt solution. The crystal's volume V increases at a rate proportional to its surface area and to the amount by which its volume is less than a limiting volume V_0 : $\frac{dV}{dt} = kx^2(V_0 - V)$, where x is the edge length of the crystal at time t . (a) Using $V = x^3$, transform the equation above to one giving the rate of change dx/dt of the edge length x in terms of x . (b) Show that the growth rate of the edge of the crystal decreases with time, but remains positive as long as $x < x_0 = V_0^{1/3}$. (c) Find the volume of the crystal when its edge length is growing at half the rate it was initially.
5. (30 pt) Please find the intergrals of the following functions:

$$(a) \int \frac{x \ln(1+x^2)}{1+x^2} dx \quad (b) \int \sin^2 x \cos^4 x dx \quad (c) \int \frac{dx}{2+e^x} \quad (d) \int \frac{\sin^3 x}{\cos^7 x} dx \quad (e) \int \frac{x+1}{\sqrt{x^2+1}} dx$$

6. (15 pt) Find the area of the surface obtained by rotating the curve of $y = e^x$ about the x -axis.