本試題是否可以使用計算機：□可使用，☑不可使用 （請命題老師勾選）

1. Please solve the following differential equations: (5 points for each one)

A. $x\dfrac{dy}{dx}+y=x^2y^2$ B. $x^2\dfrac{d^2y}{dx^2}-4x\dfrac{dy}{dx}+6y=\ln x^2$

C. $\dfrac{d^2y}{dx^2}+y=\cos^2 x$ D. $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+5y=\delta(t-2\pi)$, with $y(0)=0$ and $\left.\dfrac{dy}{dt}\right|_{t=0}=0$

2. For the following expressions, which are equal to zero ? Note that $\vec{a}$ is a constant vector, $\vec{r}(t)=x(t)\vec{i}+y(t)\vec{j}+z(t)\vec{k}$ is a position vector, $f(x,y,z)$ is an arbitrary scalar function, $\vec{F}(x,y,z)$ is an arbitrary vector function, and $\nabla=\vec{i}\dfrac{\partial}{\partial x}+\vec{j}\dfrac{\partial}{\partial y}+\vec{k}\dfrac{\partial}{\partial z}$ is the vector differential operation. (3 points for each right answer and -3 points for each wrong answer)

A. $\nabla\bullet\vec{r}$ B. $\nabla\times\vec{r}$ C. $\nabla\bullet(\vec{a}\times\vec{r})$ D. $\nabla\times(\vec{a}\times\vec{r})$ E. $\vec{a}\times(\nabla\times\vec{r})$ F. $(\vec{a}\times\nabla)\times\vec{r}$

G. $\nabla\times[(\vec{r}\bullet\vec{r})\vec{a}]$ H. $\nabla\bullet[(\vec{r}\bullet\vec{r})\vec{a}]$ I. $\nabla\times(\nabla f)$ J. $\nabla\bullet(\nabla\times\vec{F})$ K. $\nabla\bullet(\nabla f)$ L. $\nabla\times(\nabla\bullet\vec{F})$

3. Please solve the partial differential equation $\dfrac{\partial^2u}{\partial x^2}+\dfrac{\partial^2u}{\partial y^2}=0$ with the following boundary conditions: (15 points for each one)

A. $u(0,y)=0$, $u(1,y)=1-y$ for $0<y<1$ and $\left.\dfrac{\partial u}{\partial y}\right|_{y=0}=0$, $\left.\dfrac{\partial u}{\partial y}\right|_{y=1}=0$ for $0<x<1$

B. $\left.\dfrac{\partial u}{\partial x}\right|_{x=0}=0$, for $0<y<\pi$ and $u(x,0)=0$, $\left.\dfrac{\partial u}{\partial y}\right|_{y=\pi}=e^{-x}$, for $x>0$

4. For flow passing through a cylindrical tube, the pollutant concentration is increased due to the transfer from the tube into the flow at the flux of $k(C_s-C)$, where k is the transfer coefficient, C_s and C are the pollutant concentrations at the tube wall and in the flow, respectively. Please derive the relationship between the pollutant concentration C and tube length L if the pollutant concentration at inlet is C_{in}, the tube diameter is D, the volume flow rate of fluid is Q, and the flow is assumed to be plug flow. (20 points)

5. One of the numerical methods to evaluate the integral $\int_a^b f(x)dx$ is trapezoidal method, that is,

$\int_a^b f(x)dx=\dfrac{(b-a)}{2}[f(a)+f(b)]+E$, where E is the error term. Please derive the error term E in terms of *(b-a)*, *f(x)* and its derivatives evaluated at midpoint $\bar{x}=\dfrac{(a+b)}{2}$. (15 points)