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編號： 215　系所：環境工程學系甲組、乙組　　科目：工程數學

本試題是否可以使用計算機： □可使用 ， ☑不可使用 （請命題老師勾選）

1. Please solve the following differential equations: (5 points for each one)

A. $\frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=\sin 3t$ with $y(0)=0$, $\left.\frac{dy}{dt}\right|_{t=0}=0$, and $\left.\frac{d^2y}{dt^2}\right|_{t=0}=1$

B. $\frac{d^2y}{dt^2}+6\frac{dy}{dt}+5y=t-tU(t-2)$ with $y(0)=1$ and $\left.\frac{dy}{dt}\right|_{t=0}=0$

C. $\frac{d^2y}{dx^2}+y=\sec^3 x$

D. $\frac{d^2y}{dx^2}-y=x+\sin x$

2. Please derive the truncation error of the finite difference equation

$\frac{y_{i+1}-2y_i+y_{i-1}}{h^2}+p(x_i)\frac{y_{i+1}-y_{i-1}}{2h}+q(x_i)y_i=r(x_i)$ which is used for the differential equation

$\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+q(x)y=r(x)$. (15 points)

3. Please write down the computation equations explicitly if the fourth order Runge-Kutta method is used for the differential equation $y''-y(y')^2+5xy^3=x$ with initial conditions $y(0)=1$ and $y'(0)=5$. (15 points)

4. For the first-order differential equation $\frac{dy}{dx}=-2y$ with $y(0)=1$, please derive the conditions of Δx to have positive and decreasing solution for Euler's method and second-order Runge-Kutta method, respectively . (20 points)

5. In order to estimate the fugitive toluene from an open cylindrical tube of diameter D, diffusion through stagnant air of length L at steady state is assumed. Please compute the emission rate of toluene if the tube is kept at constant temperature and the toluene concentration at the interface is Cs. (15 points)

6. Please find solution for the partial differential equations $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ with following conditions: (15 points)

A. $\begin{cases} u(0,t)=0, \quad u(1,t)=0, \quad t>0 \\ u(x,0)=5\sin 2\pi x, \quad 0<x<1 \end{cases}$

B. $\begin{cases} u(0,t)=t, \quad \lim_{x\to\infty} u(x,t)=0, \quad t>0 \\ u(x,0)=0, \quad x>0 \end{cases}$