本試題是否可以使用計算機：　□可使用　，　☑不可使用　（請命題老師勾選）　　　考試日期：0301，節次：3

1. Please find the derivatives of the following functions. (18 points)

(a) $y = x^x$　(b) $y = x^2 e^x$　(c) $y = \log\left(x + \sqrt{1 + x^2}\right)$

2. Please evaluate the limit of the following functions. (7 + 8 = 15 points)

(a)　$\lim_{n \to \infty} \dfrac{(n-1)(3n+1)}{n^2 + 1}$　(b) $\lim_{n \to \infty} \sum_{k=1}^{n} \left\{ \dfrac{1}{n} \left(\dfrac{k}{n} \right)^4 \right\}$

3. Please answer the following questions.

(a) Please find the tangent plane of $z = f(x, y) = x^2 y$ at $(x, y, z) = (1, 1, 1)$. (5 points)

(b) Please obtain minimum value of　$z = 6x^2 + 9y^2 - 4xy - 8x + 6y + 3$　(8 points)

(c) Obtain the length of the arc:　$y = x^2$ $(0 \leq x \leq 1)$　(8 points)

4. Suppose there is a point P on a curve. Consider a circle that is tangent to the curve at P, and crosses with the curve at another point Q. When the point Q comes close to point P, the limit of the radius of the circle is called "radius of curvature".

(a) Express an equation for the circle which is tangent to $y = \cos x$　at $P(0,1)$, whose radius $= r$.

(3 points)

(b) Please obtain radius of curvature for $y = \cos x$　at $P(0,1)$. (8 points)

5. Please find the integrals of the following functions. (20 points)

(a) $\displaystyle \int \frac{x^4}{x^3 - x^2 - x + 1} dx$　(b) $\displaystyle \int_{0}^{\pi} x^2 \sin x\, dx$　(c) $\displaystyle \int_{1}^{e} (\log x)^2\, dx$　(d) $\displaystyle \int_{1}^{e} \frac{1 - \log x}{x^2} dx$

6. Obtain the volume of the region surrounded by the following three surfaces: 1) a surface that could be obtained by rotating $y = e^x$ around the x axis, 2) y = 1, and 3) $x = \log_e \sqrt{2}$.

(One of the fractions, which is $y \geq 1$).　(15 points)