

系所組別： 環境工程學系丙組

考試科目： 微積分

考試日期： 0307, 節次： 3

※ 考生請注意：本試題 可 不可 使用計算機

1. For a function shown below, answer the three questions. (22 points)

$$f(x) = \int_0^x \left\{ |(t-x)^2 - 1| - 1 \right\} dt$$

1-1) Please obtain $f(x)$ in a form that does not include t . (8)1-2) Derive extreme value of $f(x)$. (8)1-3) Draw the shape of $f(x)$ on a x - y coordinate when $y=f(x)$ (6)2. Obtain extreme values for the following z . (21 points)2-1) $z=x^2-3xy+2y^2$ when $x \in R, y \geq 0, x^2 + y^2 = 2$. (8)2-2) $z=\sin^3(x)+\cos^3(x)-3\sin(x)\cos(x)$ when $0 \leq x \leq 2\pi$. (8)2-3) $z = \sum_{k=1}^n (30-7k)$ when n is a positive integer number. (5)3. Obtain the volume of a shape that is made by rotating a shape around y -axis.The shape is a closed area bound from below by $y=f(x)=x^3-x^2$, and from above by x -axis.

(15 points)

4. Evaluate $\lim_{n \rightarrow \infty} a_n$ for the following arrays $\{a_n\}$. (16 points)4-1) $a_1=1, a_{n+1} = \frac{2a_n}{6a_n+1}$ (8)4-2) $a_n+S_n=n$, when $S_n = \sum_{k=1}^n a_k$ (8)

5. Please calculate the following. (18 points)

5-1) $\int_0^{\pi} \sin^5(x) \cos^6(x) dx$ (6)5-2) $\iint_D (x^2y+y^2) dx dy, D: 1 \leq x \leq 2, 2 \leq y \leq 3$ (6)5-3) $\iint_D xy^2 dx dy, D: 0 \leq y \leq x \leq 1$ (6)6. Please invert the order of integration for $\int_{-1}^1 dx \int_0^{2\sqrt{1-x^2}} f(x,y) dy$. (8 points)(HINT: invert the order means to obtain a,b,c,d for $\int_b^a dy \int_d^c f(x,y) dx$)