系所組別： 環境工程學系甲、乙組

考試科目： 工程數學　　　　　　　　　　　　　　　　　考試日期：0307，節次：3

※ 考生請注意：本試題 □可　☑不可　使用計算機

1. Please solve the following differential equations: (5 points for each one)

　A.　$y''+2y'+y = xe^{-x}$　　　　　B. $x^2y''-3xy'+3y = 2x^4e^x$

　C.　$y'''+y'' = e^x \cos x$　　　　　D.　$y''+2y'+y = xe^{-x}$

2. Finite difference method is used to solve the differential equation $y'' - 2x^2y' + (x+1)y = 5$ with boundary

conditions $y'(0) = 0$ and $y(1) = 1$.　Please determine the matrices A and B for $AY = B$ where $Y = \begin{bmatrix} y1 \\ y2 \\ y3 \\ y4 \end{bmatrix}$

and y1, y2, y3, and y4 are the values of y at x = 0, 0.25, 0.50, and 0.75, respectively. (20 points)

3. Please use the Newton forward interpolation formula to derive the computation scheme and order of accuracy if y_{i+1}, y_i, and y_{i-1} are used for the y_i'. (20 points)

4. Please solve the equation $\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}$ with the following conditions: (15 points for each one)

　A.　$\begin{cases} u(x,0) = 0, & x > 0 \\ t > 0, & u(0,t) = 1, \quad u(\infty,t) = 0 \end{cases}$　　　　B.　$-\infty < x < \infty$, and $u(x,0) = \begin{cases} 0, & x < 0 \\ e^{-x} & x > 0 \end{cases}$

5. Find the steady-state solution for $\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}$, $0 < x < 1$ and $t > 0$ with $u(x,0) = 0$, for $0 < x < 1$

and at $t > 0$, $u(0,t) = u_o$ and $\dfrac{\partial u}{\partial x}\Big|_{x=1} + u(1,t) = u_1$ (10 points).