系所組別 環境工程學系甲、乙組

考試科目 工程數學

※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. Please solve the following differential equations: (8 points for each one)

a. $(6x+1)y^2\frac{dy}{dx}+3x^2+2y^3=0$

b. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\sin x+3\cos 2x$

c. $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=\ln x^2$

d. $\frac{d^2y}{dx^2}+y=\sec^2 x$

2. Consider two concentric spheres of radius $r=a$ and $r=b$ with $a<b$ The temperature u(r) in the region between the spheres is determined from the equation $r\frac{d^2u}{dr^2}+2\frac{du}{dr}=0$, $u(a)=u_0$, $u(b)=u_1$, where u_0 and u_1 are constants. Please solve for $u(r)$. (10 points)

3. For Laplace's equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, please find solutions for the following boundary conditions. (15 points for each one)

A. $\begin{cases} u(0,y)=0, \quad u(\pi,y)=y, \quad 1<y<2 \\ u(x,1)=0, \quad u(x,2)=0, \quad 0<x<\pi \end{cases}$

B. $\begin{cases} u(0,y)=1, \quad \left.\frac{\partial u}{\partial x}\right|_{x=1}=0, \quad y>0 \\ \left.\frac{\partial u}{\partial y}\right|_{y=0}=0, \quad 0<x<\pi \end{cases}$

4. Implicit finite difference method is used for $\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}$ with $\begin{cases} T(x,0)=3x-1, \quad 0<x<1 \\ t>0, \quad T(0,t)=t^2+1, \quad \left.\frac{\partial T}{\partial x}\right|_{x=1}=0 \end{cases}$

Please derive matrices A and B if $AU=B$ where U is the unknown column matrix of $T^{n+1}{}_1, T^{n+1}{}_2, T^{n+1}{}_3$, and $T^{n+1}{}_4$, which are at x =0.25, 0.50, 0.75, and 1.0, respectively. (10 ponits)

5. For partial differential equation $\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}$ with $T(x,0)=a$ and the following boundary conditions:

a. $\begin{cases} T(t,0)=0 \\ T(t,1)=0 \end{cases}$ b. $\begin{cases} T(t,0)=b \\ T(t,1)=b \end{cases}$ c. $\begin{cases} T(t,0)=0 \\ T(t,1)=b \end{cases}$ d. $\begin{cases} T(t,0)=b \\ T(t,1)=c \end{cases}$ e. $\begin{cases} T(t,0)=0 \\ T(t,\infty)=0 \end{cases}$ f. $\begin{cases} T(t,0)=b \\ T(t,\infty)=b \end{cases}$ g. $\begin{cases} T(t,0)=0 \\ T_x(t,1)=0 \end{cases}$

h. $\begin{cases} T_x(t,0)=0 \\ T(t,1)=b \end{cases}$ i. $\begin{cases} T_x(t,0)=b \\ T(t,1)=b \end{cases}$ j. $\begin{cases} T_x(t,0)=b \\ T(t,1)=c \end{cases}$ k. $\begin{cases} T_x(t,0)=0 \\ T_x(t,1)=0 \end{cases}$ l. $\begin{cases} T_x(t,0)=T(x,0) \\ T(t,1)=0 \end{cases}$

I. Which can be solved by using $T(t,x)=H(t)R(x)$ (separation of variables) directly ?

II. For those with nonzero constant steady state solution, please write the solution.

Note that a, b, and c are constants. (答對每個 3 分,答錯每個扣 3 分)