

系所組別 環境工程學系丙組

考試科目 微積分

考試日期：0307，節次：3

※ 考生請注意：本試題 可 不可 使用計算機

## 1. Please answer the following questions. (35 points)

1-1) Please calculate indefinite integration  $F(x) = \int f(x)$  for  $f(x) = \frac{x^2+1}{x^2-3x+2}$  (5)

1-2) Please calculate indefinite integration  $F(x) = \int f(x)$  for  $f(x) = \frac{1}{x\sqrt{x^2-1}}$  (5)

1-3) Please answer the value of Gamma function  $\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt$  at  $x = \frac{1}{2}$ . (8)

1-4) Please derive the range of a positive real number  $x$  that let  $\sum_{n=1}^{\infty} \frac{1}{n^x}$  converge. (5)

1-5) Please calculate area that is surrounded by the following curves (6)

$$x^2 = py, x^2 = qy, y^2 = rx, y^2 = sx \quad \text{where } 0 < p < q, 0 < r < s$$

1-6) Please calculate the volume of a shape that is made by rotating a shape  $S$  around  $x$ -axis.  $S$  is a closed area surrounded by  $y = \cos x$ ,  $0 < x < \pi$  and  $x$ -axis. (6)2. Evaluate  $\lim_{n \rightarrow \infty} a_n$  for the following arrays  $\{a_n\}$ . (10 points)

2-1)  $a_n = \frac{4n^3 - 2n^2 + 3n - 1}{2n^3 + 3n^2 - 4n + 5}$  (5)

2-2)  $a_n = \frac{5 \times 3^n + 4 \times 4^n}{2 \times 3^n - 3 \times 4^n}$  (5)

3. For a function  $z = f(x, y) = 2x^2 + y^2 - 3xy + x - 2y + 3$ , please answer the following questions. (12 points)3-1) Please obtain the tangent plane  $g(x)$  at  $(x, y) = (2, 1)$ . (6)3-2) Please obtain a point on the surface defined by  $f(x, y)$ , at whose tangent plane is parallel with  $z = x - y + 1$  (6)

(背面仍有題目,請繼續作答)

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4. For a function  $z = f(x, y) = \frac{1}{2}xy^2 + \frac{1}{2}y^2 - 2xy - 2y$ , please answer the following questions.

(18 points)

4-1) Please calculate two stationary points  $P_1$  and  $P_2$ . ( $y$  value of  $P_1 < y$  value of  $P_2$ ). (6)4-2) Please calculate Hessian of  $f(x, y)$  at  $P_1$  and  $P_2$ . (6)4-3) Please identify whether  $P_1$  and  $P_2$  are: i) local maximal, ii) local minimal, or iii) saddle point. (6)

5. For a differential equation  $(x+y+1) dx + (x-y^2+3) dy = 0$ , please answer the following questions. (10 points)

5-1) Please demonstrate that the left hand side of this equation is closed (= a complete form). (4)

5-2) For  $f(x, y)$  and  $g(x, y)$  that satisfies  $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$ , there is a function  $F$  that satisfies  $f = \frac{\partial F}{\partial x}, g = \frac{\partial F}{\partial y}$ .

Please use this rule to solve the differential equation above. (6)

6. As shown in the figure, spherical polar coordinates utilize  $r$ ,  $\theta$ , and  $\phi$  to express a position in three-dimensional Euclidean space. On the other hand, Cartesian coordinate system uses orthogonal  $x$ ,  $y$ , and  $z$  axes. Please answer the following questions. (15 points)

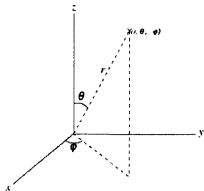
6.1) Please express  $x$ ,  $y$ , and  $z$  with  $r$ ,  $\theta$ , and  $\phi$ . (4)

6.2) To calculate the volume of an oval sphere with the following

expression, please convert  $K$  into an expression using  $r$ ,  $\theta$ , and  $\phi$ . (5)

$$\iiint_K dx dy dz,$$

$$K = \left\{ x, y, z \left| \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \right. \right\}$$

6-3) Please calculate the volume of  $K$ . (6)