

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (40 %) A second order mechanical system is described as following:

$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = f(t)$ and $\dot{y}(0) = y(0) = 0$. Answer the following questions:

- (a) Find the transfer function of above system. (5 points)
 (b) If $f(t) = \delta(t) = \text{Derac delta function}$, please find the response $y(t)$. (5 points)
 (c) If $f(t) = e^{-t} + e^t$, please find the response $y(t)$. (10 points)
 (d) If $f(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$, Please find the response $y(t)$. (10 points)
 (e) Plot (roughly) the response $y(t)$ of (d). (10 points)

2. (10%) Find the convolution integral, $\int_0^t e^{-\tau} \sin(t-\tau) d\tau$. (10 points)

3. (10%) Please explain its physical meaning of Divergence theorem of Gauss.

$$\iiint_T \text{div} F dV = \iint_S F \cdot n dA \quad (10 \text{ points})$$

4. (10%) Let $v = [yz \quad 3zx \quad z] = yzi + 3zxi + zk$

- (a) $\text{div } v = ?$ (divergence of a vector field). (5 points)
 (b) $\text{curl } v = ?$ (curl of a vector field). (5 points)
5. (10%) Find the directional derivative of $f(x, y, z) = x^2 + y^2 + z^2$ at P: (2, 2, -1) in the direction of $a = [1 \quad 1 \quad 3]$. (10 points)
6. (20%) For a matrix A, please answer the following,
- (a) Write down the definitions of eigenvector and eigenvalue (5 points)
 (b) Write down the procedure of how to obtain the eigenvalues and eigenvectors. (10 points)
 (c) What is the definition of nonsingular matrix? (5 points)