國立成功大學 106 學年度碩士班招生考試試題

系　所：生物醫學工程學系

考試科目：流體力學

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A tornado can be simply modeled as the combination of a force vortex ($V=r\omega$) and a free vortex ($V=C/r$). Note that r is the radius, C is a constant, and ω is the angular velocity. Assume the velocity reaches maximum V_{max} at $r=R$.
   (1) Derived an expression for pressure (P) in terms of distance (r) and velocity (V_{max}) in the region of free vortex. Given that the density of the air is ρ and the pressure in the infinity is P_∞. (10%)
   (2) Sketch a plot showing the relationship between the velocity and distance. (5%)
   (3) When a car is in the path of the tornado, predict if this car will be sucked in or blown away and explain why? (3%)

2. Consider using Hagen-Poiseuille's law to estimate the blood flow in a circular tube (Fig. 1). Given that the velocity profile can be expressed as a parabolic curve and the maximum velocity is 10 mm/s.

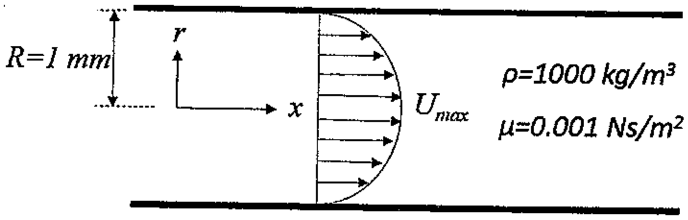

Fig. 1

$$u_x = U_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$$

   (1) Calculate the Reynolds number for the blood flow (Hint: $\mathrm{Re}=\frac{\rho U_{max} R}{\mu}$,). Is it turbulent or laminar? (7%)
   (2) Determine the wall shear stress (Hint: $\tau=\mu\left|\frac{\partial u_x}{\partial r}\right|$). (8%)
   (3) Determine whether the Bernoulli's equation can be used to calculate the pressure difference between two points at $r=R/2$ and $r=R/4$. If yes, write an expression for the pressure difference; otherwise, explain why. (Hint: rotationality) (4%)

3. In realistic situations, Bernoulli's equation can be modified with head loss terms. When fluid flows through an abrupt expansion as indicated in Fig. 2, the loss in available energy across the expansion, $loss_{ex}$, is often expressed as

$$loss_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross-sectional area upstream of expansion, A_2 = cross-sectional area downstream of expansion, and V_1 = velocity of flow upstream of expansion.

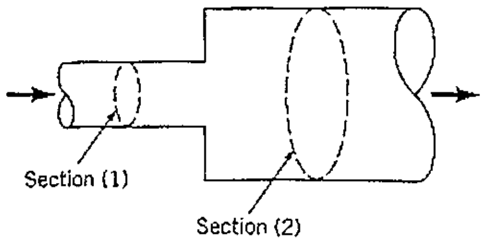

Fig. 2

(1) Explain the terms "major head loss" and "minor head loss" (8%)

(2) Derive this relationship based on the modified Bernoulli's equation. (Hint: $\frac{p_{out}}{\gamma} + \frac{V_{out}^2}{2g} + z_{out} = \frac{p_{in}}{\gamma} + \frac{V_{in}^2}{2g} + z_{in} - loss_{ex}$) (15%)

4. The window of a sight-seeing submarine is designed as the figure in Fig. 3. The width (into the paper) of the window is 1 m long. The submarine is cruising at 4 m below the sea surface.

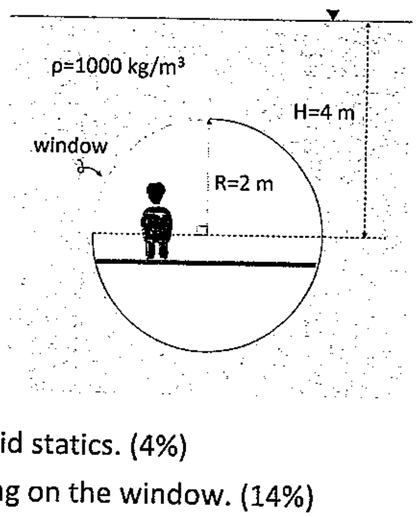

Fig. 3

(1) Explain the Pascal's law in fluid statics. (4%)

(2) Calculate the total force acting on the window. (14%)

5. Consider steady 2D flow at horizontal velocity U ($y \rightarrow \infty$) past an infinitely long and wide plate (Fig. 4). The plate is porous and there is uniform flow normal to the surface at a constant velocity V. Assume there are no pressure gradients and that gravity is negligible. The x component of Navier-Stokes equations is given below:

$$\rho\left(\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}\right)+\rho g_x$$

where μ is the dynamic viscosity of the fluid and ρ is the density of the fluid.

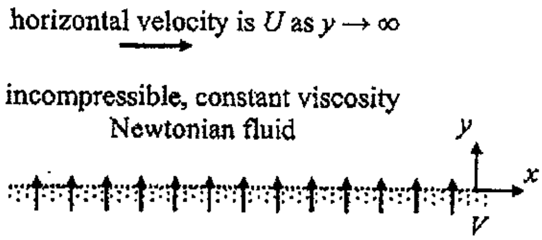

Fig. 4

(1) Prove the velocity in the x direction can be expressed as $u_x = U[1 - exp(\rho V y/\mu)]$. (18%)

(2) Explain why the vertical velocity V must be negative in this case? (4%)