

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (40 points) The function described as following; $f(t) = \begin{cases} t & 0 \leq t < 1 \\ 2-t & 1 \leq t < 2 \\ 0 & 2 \leq t \end{cases}$

Answer the following questions

- (5 points) Plot the function $f(t)$
- (10 points) Find the Laplace transformation of $f(t)$
- (10 points) Find the Fourier transformation of $f(t)$
- (15 points) If the signal $f(t)$ passing through the following elements, s , $1/s$ and e^{-2s} , plot the output signals, respectively.

Hint: $F(s) = \int_0^{\infty} f(t)e^{-st} dt$ $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$

- (10 points) Find the directional derivative of $f = x^2 + y^2 + z^2$ at $P:(2, 2, 1)$ in the direction $a = [1 \ 1 \ 1]$
- (20 points) The differential equation $\ddot{y} + 4y = \sin 2t$ and $y(0) = \dot{y}(0) = 0$
 - (10 points) Find the $y(t)$
 - (10 points) What is resonant phenomena? Please explain this phenomena from the mathematical point of view.
- (30 points) For a matrix A , answer the following:
 - (5 points) Please write down the definition of eigenvalues and eigenvectors of A
 - (10 points) Write down the procedure to get eigenvalues and eigenvectors.
 - (5 points) if $A \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$ and $A \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix}$, can you find the matrix A ? Explain your reasons.
 - (10 points) Can you find $A \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = ?$, Why? Explain your reasons.