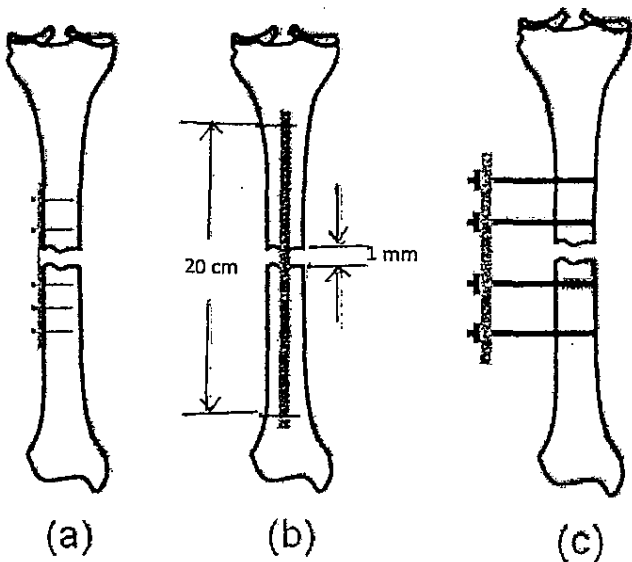


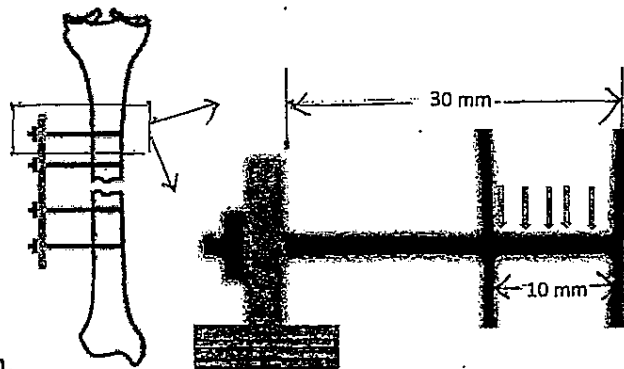
※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。  
 The common ways to repair fracture bone are internal and external fixations. The materials used for the purpose are (1) stainless steel ( $E=200\text{GPa}$ , Yielding strength=  $300\text{MPa}$ , Tensile strength= $400\text{MPa}$ ), (2) Co-Cr alloy ( $E=210\text{GPa}$ , Yielding strength=  $400\text{MPa}$ , Tensile strength= $500\text{MPa}$ ) or (3) Ti alloy ( $E=110\text{GPa}$ , Yielding strength=  $500\text{MPa}$ , Tensile strength= $600\text{MPa}$ ). It is important to maintain the stability at fracture site. In the following 2 internal fixations and one external fixation, let's discuss the following mechanics of materials questions. When people at stand phase, one leg needs support about  $1/2$  of body weight and the load may increase to 3 body weight during running. If the patient is 70 kg male. Please answer the following question.

**Internal Fixation**

**External Fixation**



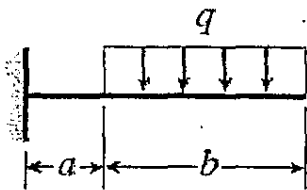
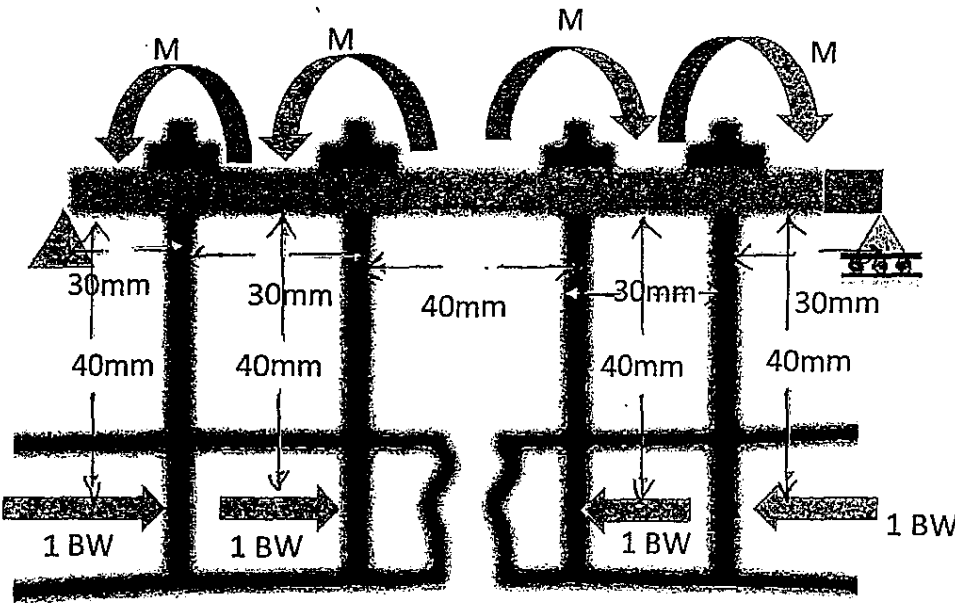
1. When the **internal fixation (b)** is used for this patient, assuming there is no friction between the intramedullary rod and bone, the distance between two connecting pins is 20 cm. The gap at fracture site is 1 mm. **What size (diameter in mm) of rod needed in order to prevent the bone fragment contact** during running phase (3 body weights) by the deformation of intramedullary rod for the materials by stainless steel and Ti alloy respectively? Comparing **internal fixation (a)** and **external fixation (c)**, please describe what characteristics of “mechanics of materials” behind these two designs? (20%)



2. For external fixation,

(A) We consider single pin's loading first. If the pin is 30 mm in length. What are the deflections at end of pin for the pin by 4 mm stainless steel and 5 mm Ti alloy during running? What's and where the principal stress on the pin? (15%)

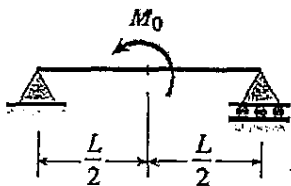
(B) Next, we consider the deflection and loading on the **Co-Cr alloy plate** (20 mm in width and 8 mm in thickness). Suppose each pin share 1 body weight (1 BW) during loading. What's the deflection at middle of plate? What's the maximal stress at the plate? ( $\sigma = Mc/I$ ;  $I = bh^3/12$  for rectangle,  $I = \pi r^4/4$  for circle) If the materials replaced by Ti alloy, what is the maximal stress at the plate? Which one (Ti or Co-Cr alloy) has **higher factor of safety** ( $n = \text{Tensile stress} / \text{stress under loading}$ ) (15%)



$$v = -\frac{qb^2}{12EI}(3L + 3a - 2x) \quad (0 \leq x \leq a)$$

$$v = -\frac{q}{24EI}(x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4) \quad (a \leq x \leq L)$$

$$I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$



$$v = -\frac{M_0 x}{24EI}(L^2 - 4x^2) \quad v' = -\frac{M_0}{24EI}(L^2 - 12x^2) \quad \left(0 \leq x \leq \frac{L}{2}\right)$$

$$\delta_C = 0 \quad \theta_A = \frac{M_0 L}{24EI} \quad \theta_B = -\frac{M_0 L}{24EI}$$

3.  $\sigma_x = 100 \text{ MPa}$ ,  $\sigma_y = -20 \text{ MPa}$ ,  $\tau_{xy} = -10 \text{ MPa}$

Using Mohr's circle, determine (a) the principal stresses and (b) the maximum shear stresses and associated normal stresses. Show all results on sketches of properly oriented elements. (C) Determine the normal and shear stresses acting on a seam oriented at an angle  $30^\circ$  of to the element, as shown in the second part of the figure. (d) Draw the Mohr's circle for (1) uniaxial tensile loading, (2) biaxial loading for  $\sigma_x > \sigma_y$  (3) biaxial loading  $\sigma_x = \sigma_y$  (4) pure shear respectively. (20%)

4. A pressurized steel tank is constructed with a helical weld that makes an angle  $\alpha = 55^\circ$  with the longitudinal axis (see figure). The tank has radius  $r = 0.6 \text{ m}$ , wall thickness  $t = 18 \text{ mm}$ , and internal pressure  $p = 2.8 \text{ MPa}$ . Also, the steel has modulus of elasticity  $E = 200 \text{ GPa}$  and Poisson's ratio  $\nu = 0.30$ .

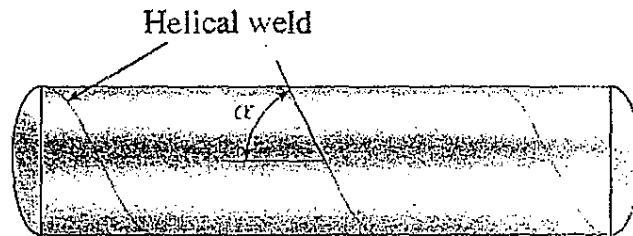
Determine the following quantities for the cylindrical part of the tank.

- (a) The circumferential and longitudinal stresses.
- (b) The maximum in-plane and out-of-plane shear stresses.

$$\epsilon_1 = \frac{\sigma_1}{2E}(2 - \nu) \quad \epsilon_2 = \frac{\sigma_2}{E}(1 - 2\nu)$$

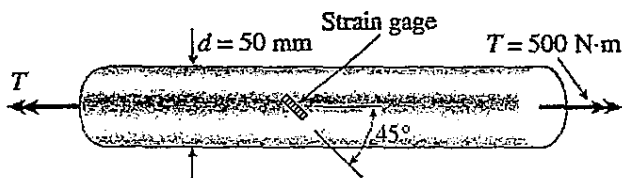
(c) The circumferential and longitudinal strains.

(d) The normal and shear stresses acting on planes parallel and perpendicular to the weld (show these stresses



on a properly oriented stress element). (20 %)

5. A solid circular bar of diameter  $d = 50 \text{ mm}$  (see figure) is twisted in a testing machine until the applied torque reaches the value  $T = 500 \text{ N} \cdot \text{m}$ . At this value of torque, a strain gage oriented at  $45^\circ$  to the axis of the bar gives a reading  $\epsilon = 339 \times 10^{-6}$ . What is the **shear modulus G** of the material? (10%)



$$\tau_{\max} = \frac{16T}{\pi d^3}$$