※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Define the following terms: (15%)
   (a) Control volume
   (b) Stokes flow
   (c) Lagrangian perspective
   (d) Kinematic viscosity
   (e) Definition of fluid

2. You are assigned to design a Cartesian diver. An upside-down tube connecting to an unknown mass m_2 is inside a plastic bottle (Fig. 1). Given that the initial volume inside the tube is V_1, the volume of the unknown mass is V_2, and the mass of the tube is m_1. **(a)** What is the tension of the cable between the tube and the unknown mass? **(b)** What is the maximum mass (m_2) that the tube can bear in order to float at the top of the bottle? Express the mass in terms of m_1, V_1, and V_2. **(c)** Why does the tube drops after the bottle is deformed by an external force? (20%)

3. An ideal fluid of density ρ is flowing at a uniform velocity of U in a straight pipe flow (Fig. 2). Assume the potential difference is negligible.
   (a) list the four assumptions necessary for the Bernoulli equation. (6%)
   (b) Derive an expression for the pressure at B when the pressure at A is P_0. (10%)
   (c) Show that the Bernoulli equation fails when the velocity profile turns parabolic due to viscosity. (Hint: vorticity=0; $u_{Ax} = U[1-\left(\frac{2y}{H}\right)^2]$) (8%)

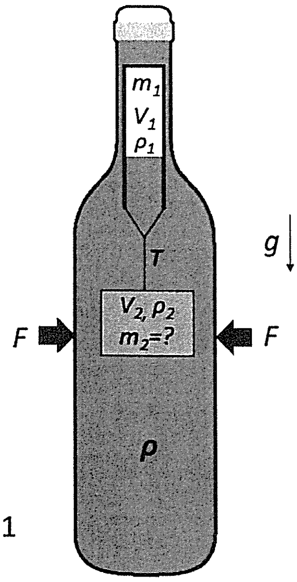

Fig. 1

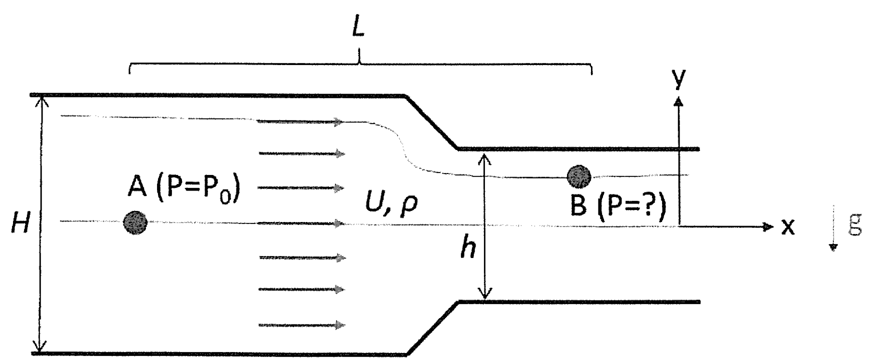

Fig. 2

4. In a uniform flow, a control volume is drawn as shown in Fig. 3.

   (a) Show the mass fluxes through surface A and surface B are identical (by conservation of mass). (10%)

   (b) Why is there no difference between both of the mass fluxes? (7%)

5. An incompressible, Newtonian liquid of density ρ and dynamic viscosity μ is sheared between concentric cylinders as shown in Fig. 4. The inner cylinder radius is R_i and the outer cylinder radius is R_o. Determine the velocity profile (u_θ) for the liquid in the gap when the inner cylinder rotates at a constant angular speed of ω. All boundaries are in no-slip conditions and gravity is perpendicular to the paper. The continuity and Navier-Stokes equations in cylindrical coordinates are given below. (24%)

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}-\frac{u_r u_\theta}{\partial r}+u_z\frac{\partial u_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)-\frac{u_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right]+\rho g_\theta$$

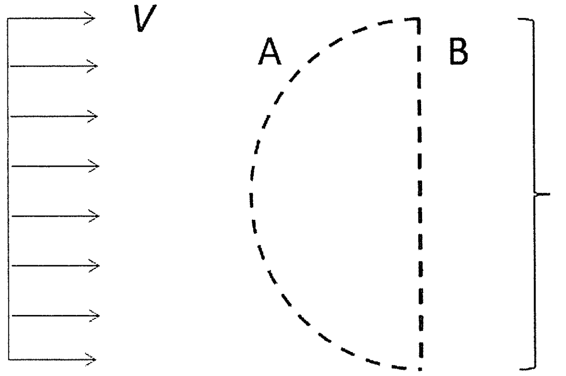

Fig. 3

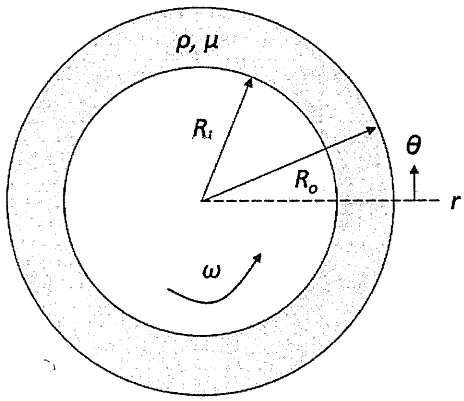

Fig. 4