國立成功大學109學年度碩士班招生考試試題

系　　所：生物醫學工程學系

考試科目：工程數學

考試日期：0210，節次：1

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20 points)The function described as following; $f(t) = u(t) - u(t-1)$
   $u(t)$ is a unit step function
   Answer the following questions
   (a) (5 points) Find the Laplace transformation of f(t)
   (b) (5 points) Find the Fourier transformation of f(t)
   (c) (10 points) Let f(t) be a periodical function with four seconds period ($p = 4$), then find its Fourier series. (Hint: even or odd expansion)

   (Hint: $F(s) = \int_0^\infty f(t)e^{-st}dt$　　$F(\omega) = \int_{-\infty}^\infty f(t)e^{-i\omega t}dt$

2. (10 points) Find the following integration

   $$\int_0^t \tau \cos(t-\tau)\, d\tau$$

   (Hint: Using Laplace transform techniques)

3. (10 points) Find the directional derivative of $f = x^2 + y^2 + z^2$
   at P:(3, 2, 1) in the direction $a = [1 \quad 0 \quad 1]$

4. (20 points)　To solve the differential equation $\ddot{y} + 4\dot{y} + 8y(t) = sin2t$
   $y(0) = \dot{y}(0)=0$

5. (20 points) $x_1(n) = [1,2,3,1]$, $x_2(n) = [1,2,3,1,0,0,0,0]$
   $x_3(n) = [0,0,0,0,1,2,3,1]$
   The DFT (Discrete Fourier Transform) is

   $$X(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{i2\pi kn}{N}}$$

   a. (10 points) Find the DFT of $x_1(n)$, $X_1(k)$
   b. (10 point) Show the differences of $X_2(k)$ and $X_3(k)$
   The $X_2(k)$ and $X_3(k)$ are the DFT of $x_2(n)$ and $x_3(n)$, respectively.

6. (20 points)　A matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$

   a. (10 points) Find the eigenvalues and eigenvectors of A.
   b. (10 points) Find the inverse matrix of A.