國立成功大學 109 學年度碩士班招生考試試題

系　所：生物醫學工程學系

考試科目：流體力學

考試日期：0210，節次：2

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. In hemodynamics, blood vessel is typically treated as a rigid and hollow cylinder for simplicity (Fig. 1).

(a) Given that flow is governed by the Navier-Stokes equations (see below), **derive an expression for the velocity profile (u_z)** with appropriate assumptions. (Notice: the assumptions and boundary conditions must be addressed) (10%)

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

(b) **Prove** the Hagen Poiseuille equation ($Q=\frac{\pi R^4 \Delta P}{8\mu L}$) using the derived velocity profile and **address the relationship** between flow rate and cylindrical diameter according to the equation. (Notice: μ denotes the viscosity of fluid, Q is the volumetric flow rate, R is the radius of the blood vessel, L is the length of the blood vessel, and ΔP is the pressure difference between both ends of L) (10%)

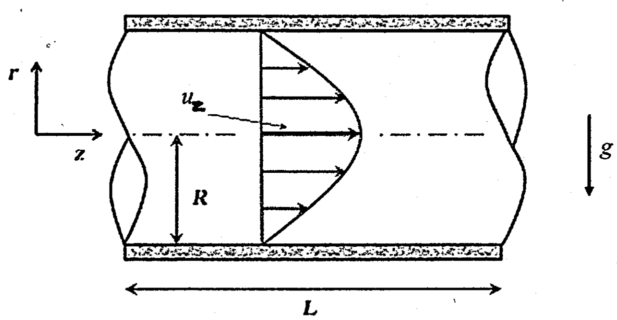

Fig.1

2. Bernoulli Equation defines an ideal relationship between flow work, potential energy, and kinetic energy without energy loss.

(a) **Write down the assumptions** for the Bernoulli Equation. (6%)

(b) Given that the structure of a tornado can be simplified as a combination of a forced ($V=r\omega$) and a free ($V=K/r$) vortices (Fig. 2), **derive an expression** for the pressure difference between point 1 ($r=0$)and point 3 ($r\to\infty$). Note that V is the wind velocity, r is the radius of the tornado, ω is the angular velocity, K is a constant, ρ is the density of the fluid, and R is the maximum radius of the force vortex located at point 2. (10%)

(c) **Explain** what will happen if a person stands near the tornado according to the expression derived in (b). (2%)

(Hint: for a forced vortex, the equation is $p+\rho\int\frac{V^2}{R}dn+\gamma z=Constant$)

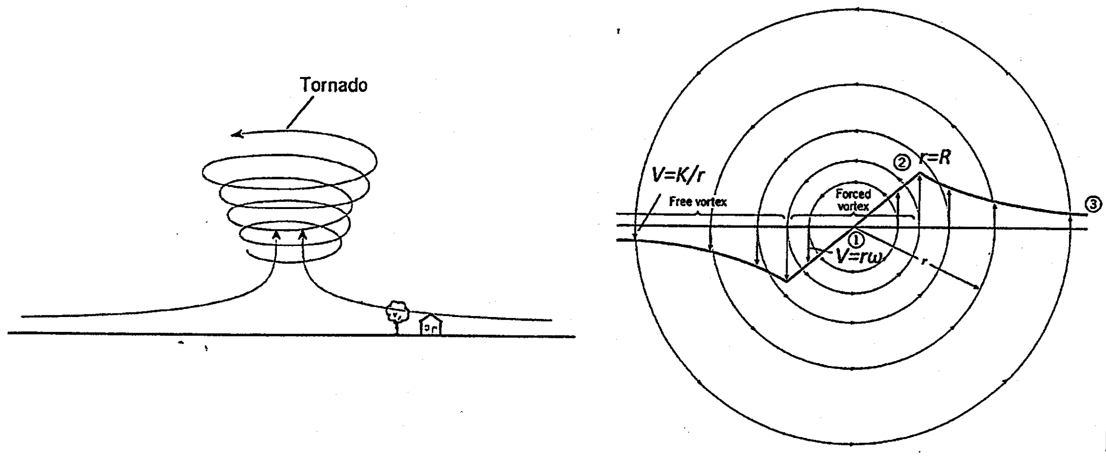

Fig. 2

3. A siphon phenomenon is used to suck water from a water tank and then flow to a lower reservoir (see Fig. 3.).

(a) **Draw the Hydraulic Grade Line (HGL)** (i.e. graphical Bernoulli Equation) for the hose. (Notice: Annotate the relationship of the three terms on the graph.) (6%)

(b) **Explain** whether water will remain in the hose or come out at the hole H based on the graphical Bernoulli Equation? (2%)

(c) If the minor losses at A (entrance), B (expansion), C (exit) and the major losses of the hose are considered, **determine the velocity of the free jet at the exit of the hose**. Assume the lengths of the both sections (A->B and B->C) of the hose are 1-m long each; the diameters of D_1 and D_2 are 0.1 m and 0.2 m, respectively. The viscosity and density of water are 10^{-3} Pa·s and 1000 kg/m^3, respectively. The gravitational acceleration is 10 m^2/s. The velocity at B is the same as that at A. (12%)

(Hint: $h_{major} = 0.02\frac{l}{D}\frac{V^2}{2g}$; $h_{minor} = K\frac{V^2}{2g}$; $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{major} + h_{minor}$)

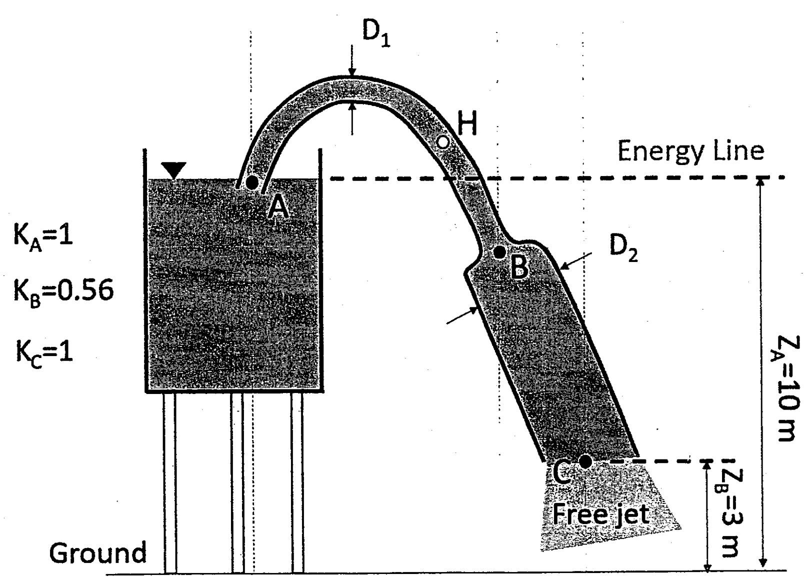

Fig. 3

4. A cart with a total weight W is pushed by a horizontal water jet at a constant velocity V_{jet} but stops on a scale (Fig. 4.). Determine, at this moment,

(a) the output jet velocity V_{out} (Notice: A **Control Volume** must be drawn), (6%)

(b) the **friction force *F*** acting on the cart, and (8%)

(c) the **reading of the scale**? (Notice: FBD is required.) (8%)

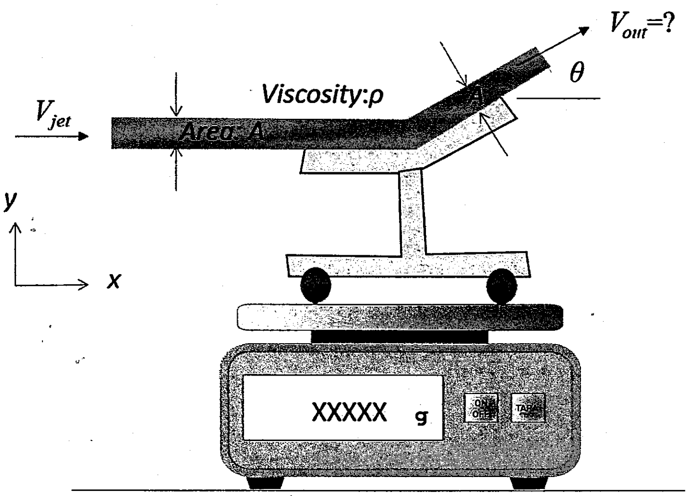

Fig. 4

5. Streamlines, streaklines and pathlines are three common measures used to delineate kinematics of flow.

(a) Explain at what condition the three lines will coincide with each other? (4%)

(b) Briefly explain the definitions for streamlines, streaklines, and pathlines. (6%)

(c) The kinematic equation of the flow is written as follows. Derive the streamlines of the flow and draw its flow pattern in the Cartesian coordinate system. (10%)

$$U = \frac{-V_0 y}{\sqrt{x^2+y^2}}\vec{i} + \frac{V_0 x}{\sqrt{x^2+y^2}}\vec{j}$$