國立成功大學

111學年度碩士班招生考試試題

編　號：160

系　所：生物醫學工程學系

科　目：工程數學

日　期：0219

節　次：第 1 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15 %) $f(t) = u(t) - u(t-1)$, $u(t)$ is a unit step function. The function f(t) is a non-periodic function, then f(t) can not represented by Fourier series. Therefore, we can utilize the odd half-range expansion to become a periodic function, $p = 4$, find its Fourier series.

2. (20 %)　f(t)=g(t)*h(t), * is convolution operation
   a. (10 %)Write down the integral expression of function f(t)
   b. (10 %)If g(t)=u(t), unit step function, h(t)=cost , then find f(t).

3. (20 %) The function $f(x, y. z) = x^2 + 2y^2 + 3z^2$
   a. (5 %) Find the ∇f (grad f)
   b. (5 %) Find the $\nabla \cdot \nabla f$ (Div ∇f)
   c. (5 %) Find the $\nabla x \nabla f$ (Curl ∇f)
   d. (5 %) Find the directional derivative $D_b f$
      at P:(1, 2, 3) in the direction $b = [1 \quad 1 \quad 1]$

4. (25 %)　A matrix $A = \begin{bmatrix} 3 & 5 \\ 5 & 3 \end{bmatrix}$
   a. (10 %) Find the eigenvalues and eigenvectors of matrix A.
   b. (5 %) Find the inverse matrix A^{-1}
   c. (10 %) $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, if $x_1{}^2 + x_2{}^2 = 1$
      then $a \le y_1{}^2 + y_2{}^2 \le b$, find a and b

5. (20%)　The differential equation $\ddot{y} + 6\dot{y} + 13y = cos3t$
   a. (10%) When $y(0) = \dot{y}(0) = 0$, then find y(t).
   b. (10 %) When $\ddot{y} + 6\dot{y} + 13y = 0$, $y(0) = -1, \dot{y}(0) = 0$　plot the homogeneous solution y(t) roughly without any calculation and explain your reasons.