國立成功大學

114學年度碩士班招生考試試題

編　號：112

系　　所：生物醫學工程學系

科　　目：工程數學

日　　期：0210

節　　次：第 1 節

注　　意：1.不可使用計算機
　　　　　2.請於答案卷(卡)作答，於
　　　　　　試題上作答，不予計分。

Please write the detailed solutions to the following questions.

1. (35 pts) Solve the following differential equations.
    (1) (15 pts) $[D^3 + D^2 + 2D - 4I]y = x\cos 2x + xe^x$
    (2) (20 pts) $[D^2 + 3D + (2 + \lambda)I]y = 0$, $y(-1) = y(1) = 0$.

2. (25 pts) Let f and g be functions with continuous first and second order partial derivatives on a region D bounded by a smooth surface S. Show that
    (1) (15 pts)

$$\iint_S (f\nabla g)\,d\vec{\sigma} = \iiint_D (f\nabla^2 g + \nabla f \cdot \nabla g)\,dV$$

    (2) (10 pts)

$$\iint_S (f\nabla g - g\nabla f)\,d\vec{\sigma} = \iiint_D (f\nabla^2 g - g\nabla^2 f)\,dV$$

3. (25 pts) Solve the partial differential equation:

$$U_{tt}(x,t) = 15U_{xx}(x,t), \quad \text{for } x \in \mathbb{R}, \ t > 0$$
$$U(x,0) = \cos\pi x, \quad U_t(x,0) = e^{-2x}\sin x, \quad \text{for } x \in \mathbb{R}$$

4. (15 pts) The Fourier Transform are defined as follows:

$$F(\omega) = \mathcal{F}[f] = \int_{-\infty}^{+\infty} f(t)e^{-iwt}dt,$$

    $F(\omega)$ is also called the frequency response of the signal $f(t)$. Suppose we have a system

$$y'(t) + 5y(t) = \int_{-\infty}^{+\infty} x(v)z(t-v)dv - 3x(t),$$

    where $z(t) = \cos(2t) + 5\delta(t)$.

    (1) (8 pts) Find the frequency response of the system, i.e., $H(\omega) = \frac{Y(\omega)}{X(\omega)}$.

    (2) (7 pts) Determine the impulse response of the system, i.e., $h(t)$.