國立成功大學 78 學年度 醫學工程研究所 入學考試(工程數學試題) 共 1 頁 第 1 頁

(甲·乙組)

1. Four variables u, v, x, y are related by equations
(12%) $u^2 - v^2 + 2x = 0$ and $uv - y = 0$
Consider x, y are independent variables. Find $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial^2 u}{\partial x^2}$ in terms of u, v.

2. Find the dimensions of the rectangular box, without a top,
(12%) of maximum capacity whose surface is $108\ cm^2$.

3. For the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$, use $y \equiv \sum_{n=0}^{\infty} A_n x^n$
(12%) as a solution, find the relation between the Coefficients A_n's.

4. If $f(t)$ is periodic with period a, show that its Laplace
(13%) transform is

$$\mathcal{L}[f(t)] = \frac{1}{1-e^{-as}} \int_0^a f(t) e^{-st} dt$$

5. Given partial differential equation $ap + bq + cz = 0$ where
(13%) a, b, c are constants and $p \equiv \frac{\partial z}{\partial x}$, $q \equiv \frac{\partial z}{\partial y}$
Find the general solution.

6. Given Cartesian force vector $\vec{F} \equiv (4xy - 3x^2z^2)\vec{i} + 2x^2\vec{j} - 2x^3z\,\vec{k}$.
(12%) Is this force field solenoidal? Is the force field conservative?, find the corresponding scalar potential and the work done by moving a partical from $(\frac{\pi}{2}, -1, 2)$ to $(0, 1, -1)$.

7. For analytic complex function $f(z) \equiv u(x,y) + i\,v(x,y)$,
(13%) express Cauchy-Riemann equations in Cartesian coordinates form into polar coordinates form.

8. Two matrices $A \equiv \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$ and $B \equiv \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$. if they are
(13%) commutative so that $AB = BA$. what are the necessary conditions to be satisfied by their elements? If A is given, can matrix B uniquely determined?