1. Consider a system shown in Fig. 1.

(a) Find the closed-loop transfer function.

(b) Determine the range of K so that the system is stable.

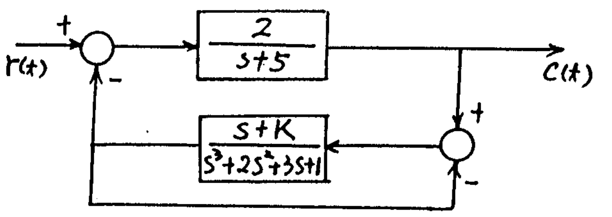

Fig. 1

2. A discrete-data system is characterized by the transfer function

$$\frac{C(z)}{R(z)} = \frac{2z}{z^2 - z + K}$$

(a) Write the dynamic equation for the system in vector-matrix form.

(b) Determine the range of K so that the system is stable.

3. A unity feedback control system has an open-loop transfer function

$$G(s) = \frac{K}{(s^2-1)(s+4)^2}.$$

Sketch the root locus ($0 \le K < \infty$) diagram of the system.

4. A closed-loop system is shown in Fig. 2

(a) For $K = 20$, determine the value of a and b to give an overshoot of 10 percent and a time constant of 0.2 sec of the system response to a unit step input. Time constant is defined here as the inverse of the damping factor.

(b) If the value of K is increased slightly, how does it affect the damping ratio of the system. (5%)

(c) Find the steady-state error due to a unit parabolic input. (K, a, and b coefficients are given in part a) (5%)

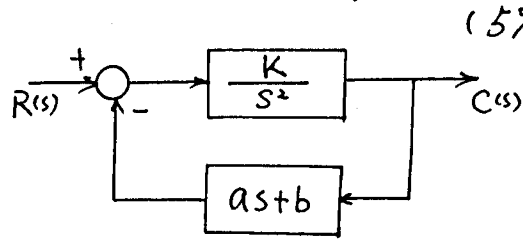

Fig. 2

5. Consider that a linear time-invariant SISO system is described by the following dynamic equations:

state equation : $\dot{x}(t) = A\,x(t) + B\,u(t)$

output equation : $c(t) = D\,x(t)$.

where A is diagonal with entries $\{\lambda_i\}$.

(a) Show that the system is state controllable if and only if the λ_i are distinct and all components of B are nonzero.

(b) Show that the system is observable if and only if the λ_i are distinct and all components of D are nonzero.