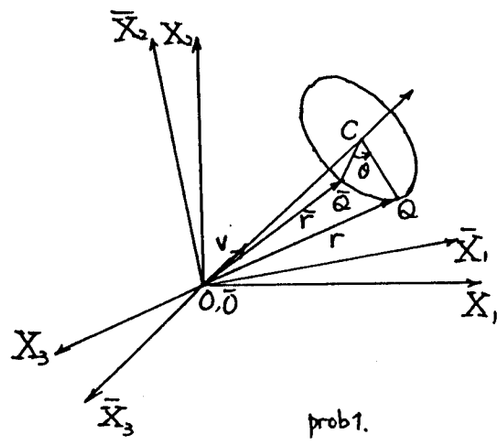
1. (20%) The origins of the two coordinate systems $\bar{X}_1\bar{X}_2\bar{X}_3$ and $X_1X_2X_3$ coincide as shown in Figure. Assume that the axes of these two systems are initially parallel. Let $\bar{\mathrm{r}}$ be the position vector of point $\bar{Q}$ whose coordinates are assumed to be fixed in the $\bar{X}_1\bar{X}_2\bar{X}_3$ coordinate system. Let the reference $\bar{X}_1\bar{X}_2\bar{X}_3$ rotate an angle θ about the axis OC. As the result of this rotation, point $\bar{Q}$ is translated to point Q. The position vector of point Q in the $X_1X_2X_3$ coordinate system is denoted by the r. v is a unit vector along the axis of rotation OC. Derive the transformation that defines the relative orientation between these two coordinate systems,

$$\mathrm{r} = \bar{\mathrm{r}} + (\mathrm{v}\times\bar{\mathrm{r}})\sin\theta + 2[\mathrm{v}\times(\mathrm{v}\times\bar{\mathrm{r}})]\sin^2\frac{\theta}{2}.$$



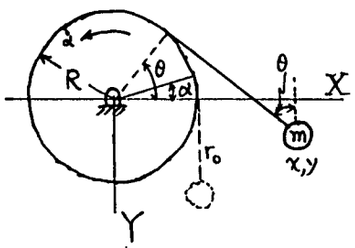
prob1.

2. (20%) Let us suppose that the disk can be given by any desired rotational motion by a motor (not shown) attached to its axis. The string to which m is attached is wrapped around and fastened to the disk. The angular position of this disk is given by α, measured from the fixed X axis to a line drawn on the face of the disk. Angular position of the string (assumes always to be tangent to the pulley) is given by θ. r_0 is the initial length of the string as indicated. For mass m,

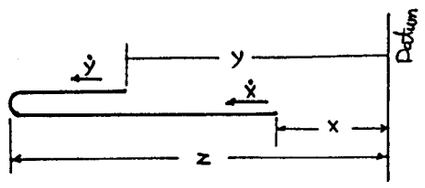
   a. What is the kinetic energy T?

   b. What is the generalized force F_θ?

   c. Construct Lagrange's equation of motion in terms of θ.



prob. 2.



prob. 3

3. (20%) We realize the whip as an inextensible uniform string with no bending stiffness. Let it be arranged in a straight line as in Figure. It is double in the manner shown. The distance of one end of the string from some fixed datum is x, and its velocity is $\dot{x}$, that of the other end is y, and its velocity is $\dot{y}$. The kink is a distance z from the datum and the rope is of length l. The kinked string is initially in a straight line and the velocities $\dot{x}$ and $\dot{y}$ also lie in this line. We are given

$$x = x_0 - vt,$$

where $v > 0$ is a constant. Discuss the motion; in particular, explain what happens when the string straightens out.

4. (20%) Two wires are attached to vertical posts, as shown in Fig. 4. Assuming that the wires are straight, determine the value of h for which the shortest distance between the wires is one foot.

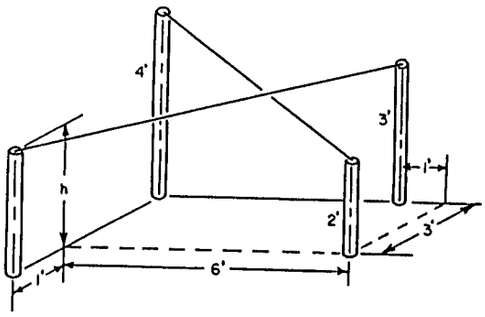


Fig 4.

5. (20%) A thirty-four pound weight is supported by a cable which is wound on a fixed horizontal drum and is attached to a wall, as shown in Fig. 5, The coefficient of friction for the drum and cable has the value 0.2. Determine the reaction of the cable on the wall.

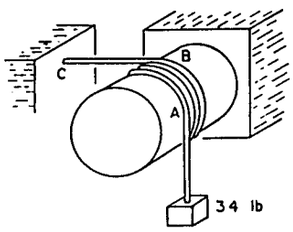


Fig.5.

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