(甲、乙)

國立成功大學79學年度醫學工程研究所入學考試(工程數學試題) 共1頁 第1頁

1. (12%) Find the general solution of the simultaneous equations

$$\begin{aligned}\frac{dx}{dt}-\frac{dy}{dt}+x &= \cos t\\ \frac{d^2x}{dt^2}-\frac{dy}{dt}+3x-y &= e^{2t}\end{aligned}$$

2. (12%) Determine a unit vector normal to the surface $x^3-xyz+z^3=1$ at the point (1,1,1).

3. (12%) If ψ is any scalar field, apply the divergence theorem to prove that

$$\iiint_{\mathcal{R}} \nabla\psi dV = \iint_{\mathcal{L}} \psi \mathbf{n} dS,$$

where $\mathcal{R}$ is any region, $\mathcal{L}$ is its boundary surface and $\mathbf{n}$ is the unit outward normal to $\mathcal{L}$.

4. (12%) Find the Fourier series of the function defined as

$$g(x)=\begin{cases} x+\pi & \text{for } 0\le x<\pi \\ -x-\pi & \text{for } -\pi\le x<0 \end{cases} \quad \text{and } g(x+2\pi)=g(x)$$

5. (12%) Prove that if $\mathbf{S}$ is any anti-symmetric matrix, then $(\mathbf{I}-\mathbf{S})(\mathbf{I}+\mathbf{S})^{-1}$ is orthogonal.

6. (12%) Use *residue calculus* to evaluate the integral

$$\int_0^\infty \frac{\cos mx}{(x^2+a^2)(x^2+b^2)}dx$$

7. (12%) Use the *complex inversion integral* to find the inverse Laplace transform of the function

$$F(s)=\frac{6s^2+10s+2}{s(s+1)(s+2)}$$

8. (16%) Find the solution of Laplace's equation $\nabla^2 T=0$ in the rectangular region $o\le x\le a$, $0\le y\le b$ subject to the boundary conditions

$$\frac{\partial T}{\partial x}(0,y)=0,\quad \frac{\partial T}{\partial x}(a,y)=0,$$
$$T(x,0)=x(a-x),\quad T(x,b)=0.$$